$\boxed{\text{Introduction to } \cancel{\text{OFPD's}} \text{ GPD's}}$

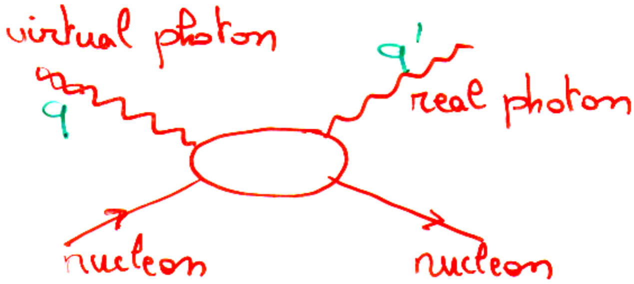
(Generalized Parton Distributions)

in the context of

$\boxed{\text{Deeply virtual Compton Scattering (DVCS)}}$

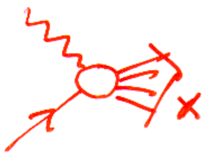
(eventhough Meson Production is interesting too)

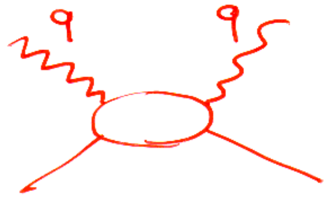
WHAT IS VIRTUAL COMPTON SCATTERING? (VCS)

1) VCS = 

$q'_\mu q'^\mu = 0$

2) looks like inclusive DIS

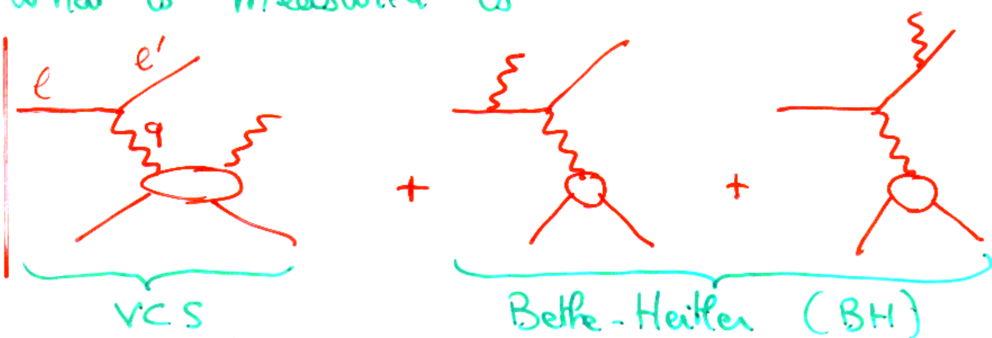
$\sigma_{DIS} \sim \sum_X \Big|$ 

$\Big|^2 \sim \mathrm{Im}$ 

(optical theorem)

3) but

- $q' \neq q$ in VCS
- what is measured is

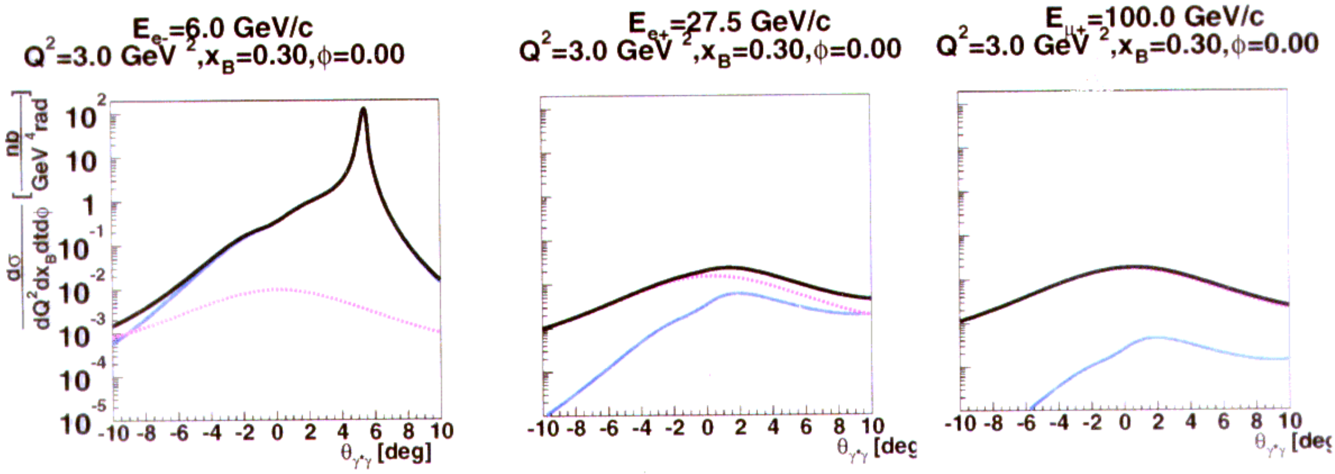
$\sigma_{ee'\gamma} = \Big|$ 

$\Big|^2$

$= |T^{VCS}|^2 + |T^{BH}|^2 + 2\,\mathrm{Re}\left(T^{VCS}\, T^{BH\,*}\right)$

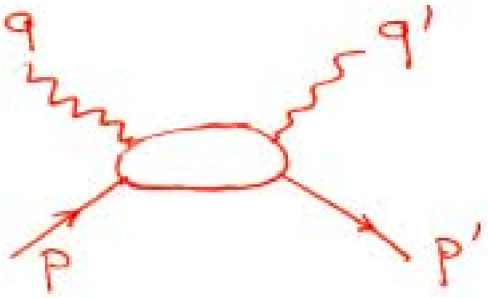
with :

- T^{BH} calculable
- $|T^{BH}/T^{VCS}|$ tunable by beam energy

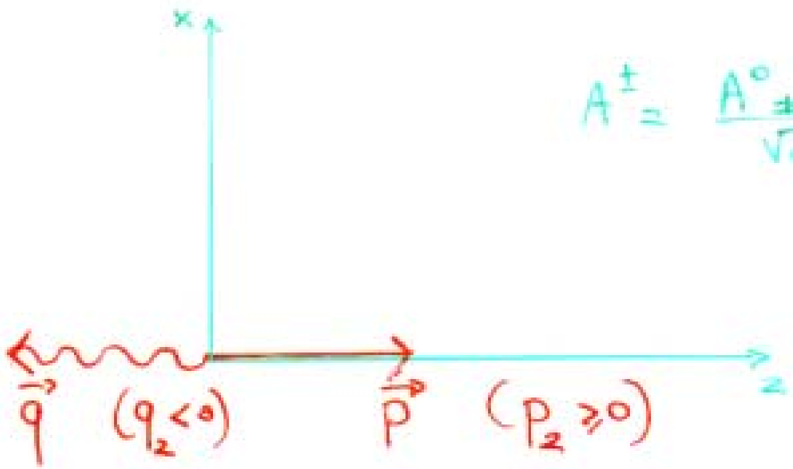


E_{e-}=6.0 GeV/c
Q^2=3.0 GeV^2,x_B=0.30,ϕ=0.00
E_{e+}=27.5 GeV/c
Q^2=3.0 GeV^2,x_B=0.30,ϕ=0.00
$E_{\mu+}$=100.0 GeV/c
Q^2=3.0 GeV^2,x_B=0.30,ϕ=0.00
$\frac{d\sigma}{dQ^2 dx_B dt d\phi}$ $\left[\frac{nb}{GeV^4 rad}\right]$
$\theta_{\gamma^*\gamma}$ [deg]

Deep VCS (generalized Bjorken regime)



- $Q^2 = -q_\mu q^\mu \longrightarrow \infty$
- $x_B = \dfrac{Q^2}{2p.q}$ finite ($s \sim Q^2$: 1 single large scale)
- $t = (p' - p)^2$ finite ($\ll Q^2$)

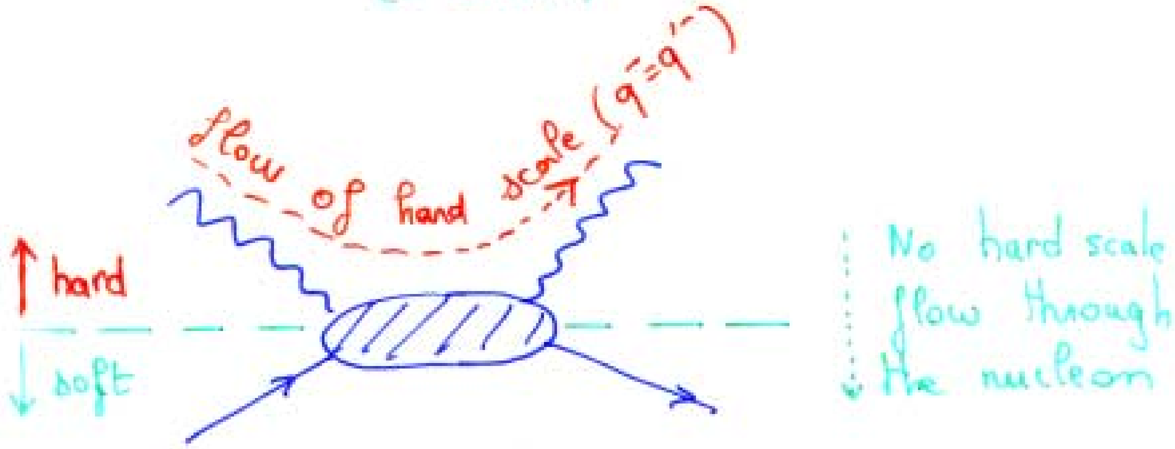


$$A^\pm = \frac{A^0 \pm A^z}{\sqrt{2}}$$

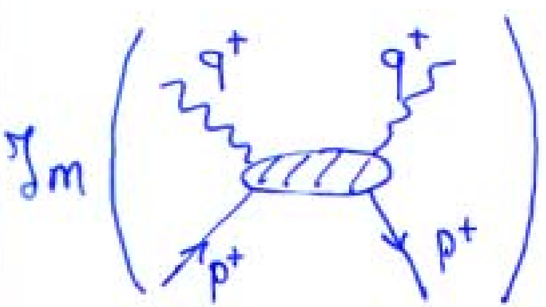
Do the kinematics in the Bjorken limit

$q^+ = -x_B\, p^+$	$q_\perp = 0$	$q^- = Q^2/2x_B p^+$
$q'^+ = 0$	$q'_\perp = \sqrt{(1-x_B)(M^2 x_B - t)}$	$q'^- = Q^2/2x_B p^+$
Soft	Soft	Hard

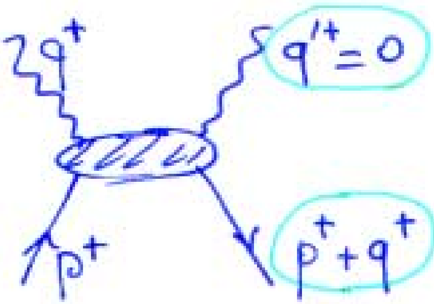
(t small!)



$\boxed{\text{Compare DIS and DVCS}}$



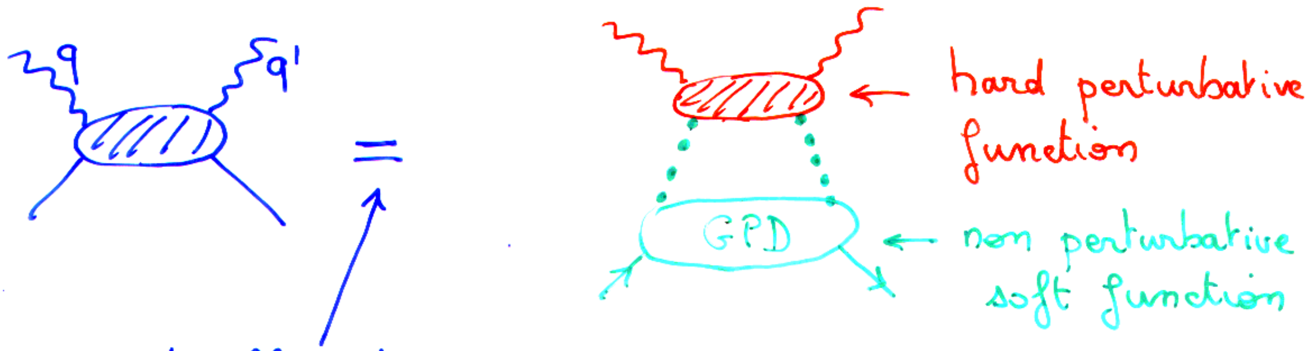
DIS



DVCS

$\boxed{\text{QCD factorization}}$ (Collins-Freund / Ji-Osborne / Radyushkin, 1998)

$(Q^2 \to \infty \quad x_B \text{ finite} \quad t \sim M^2)$ $(q'^2 \text{ arbitrary})$



to all orders in α_s

$$+ \, O(1/Q)$$

(at leading power of $1/Q$ only 2 partons are exchanged)

<u>Generic form of factorization:</u>

$$T_{DVCS} = \int dx \, H^{hard}(x, x_B, Q^2, \mu^2, \alpha_s(\mu^2)) \, S^{soft}(x, \mu^2, \alpha_s(\mu^2), P, P')$$

To make the perturbative function calculable:

choose $\mu^2 \sim Q^2$

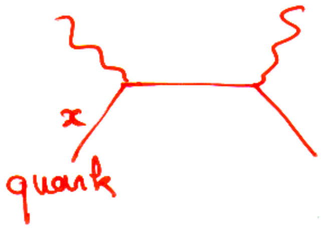
$\longrightarrow$ $\alpha_s(Q^2)$ hopefully small enough

$\longrightarrow$ suppress $\ln \mu^2/Q^2$ in H^{and}.

$\longrightarrow$ S^{oft} becomes function of Q^2, evolution...

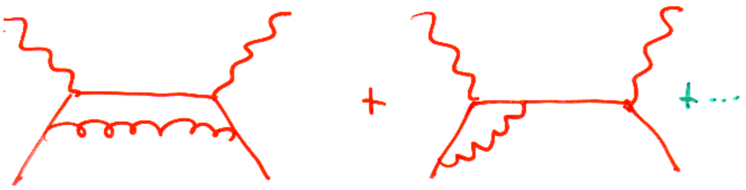
<u>Expand H^{and} in $\alpha_s(Q^2)$</u>

$(\alpha_s)^0$

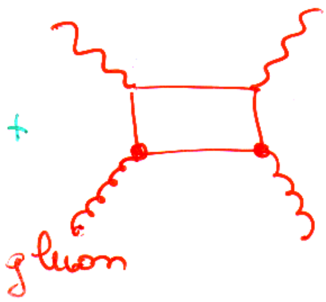


quark hand bag
$\left(\sim \dfrac{1}{x - x_B + i\varepsilon}\right)$

$(\alpha_s)^1$:



corrections to quark handbag

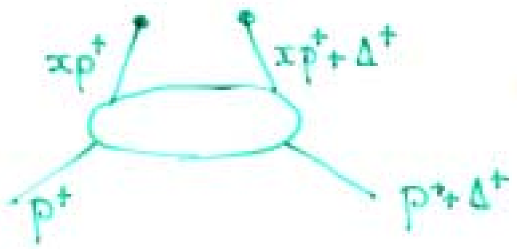


gluon handbag

$$S^{oft} = \int dy^- \, e^{i(xp^+)y^-} \langle p' | \bar{\Psi}(0)\, \gamma^+ \, \Psi(0^+, y^-, 0_\perp) | p \rangle$$

(or $\gamma^+\gamma^5$ or gluon operator)

$=$



$\Delta = p' - p$

Notice: S^{oft} is defined independently of the DVCS reaction

in DVCS $\Delta^+ = -x_B \, p^+$, in DIS $\Delta^+ = 0$

→ "skewedness" parameter $\boxed{-\dfrac{\Delta^+}{p^+} = \zeta}$ (dzeta)

(or $-\dfrac{\Delta^+}{p^+ + \Delta^+/2} = \xi$ (ksi))

$$T^{DVCS} \sim \int dx \, \frac{1}{x - x_B + i\varepsilon} \, S^{oft}(x, \zeta = x_B)$$

$$\sigma_{DIS} \sim \mathrm{Im} \int dx \, \frac{1}{x - x_B + i\varepsilon} \, S^{oft}(x, \zeta = 0)$$

$$= S^{oft}(x_B, \zeta = 0)$$

Parametrization of S^{soft}: GPD's (Ji 1996)

$$\int dy^- \, e^{i(xp^+)y^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(0, y^-, 0_\perp) | p \rangle$$

$$= \bar{u}(p') \left[h(x,\zeta,t)\gamma^+ + i\, e(x,\zeta,t) \frac{\sigma^{+\nu}\Delta_\nu}{2m} \right] u(p)$$

$$\left(+ \quad \gamma^+\gamma^5 \rightarrow \tilde{h}, \tilde{e} \quad + \text{gluons} \rightarrow h_G \; e_G \right)$$

Orthodoxy is $H(x, \xi, t) = h\left(\underbrace{x \frac{p^+}{p^+ + \Delta^+/2}}, \xi \frac{p^+}{p^+ + \Delta^+/2}, t\right)$

fraction with respect to $\frac{p+p'}{2}$

fraction with respect to p

At leading order in $1/Q$ one needs

4 GPD's H, E, $\tilde{H}$, E (or $h, e, \tilde{h}, \tilde{e}$)

for each parton.

Unifying aspects of GPD's

① forward limit: $\zeta = 0 \quad t = 0$

$$h(x,0,0)\,\bar{u}(p)\gamma^+ u(p) = \int dy^- \, e^{i x p^+ y^-} \langle p | \bar{\Psi}(0)\gamma^+ \Psi(0,y^-,0_\perp) | p \rangle$$

$$\rightarrow \boxed{\begin{array}{l} h(x,0,0) = q(x) \\ \tilde{h}(x,0,0) = \Delta q(x) \end{array}} \qquad \text{(ordinary pdf)}$$

N.B.: $e(x,0,0)$ and $\tilde{e}(x,0,0)$ exist but never appear in DIS because $p = p'$

② Sum rules: $\int dx \ldots \rightarrow \delta(y^-)$

$$\rightarrow \langle p' | \bar{\Psi}(0)\gamma^+ \Psi(0) | p \rangle = \text{current}$$

$$\boxed{\begin{array}{ll} \int dx\, h(x,\zeta,t) = F_1(t) \; ; & \int dx\, e(x,\zeta,t) = F_2(t) \\ \int dx\, \tilde{h}(x,\zeta,t) = g_A(t) \; ; & \int dx\, \tilde{e}(x,\zeta,t) = h_A(t) \end{array}}$$

Interpretation (M. Burkhardt):

GPD(x,ζ,t) = contribution of the parton carrying x to the form factor

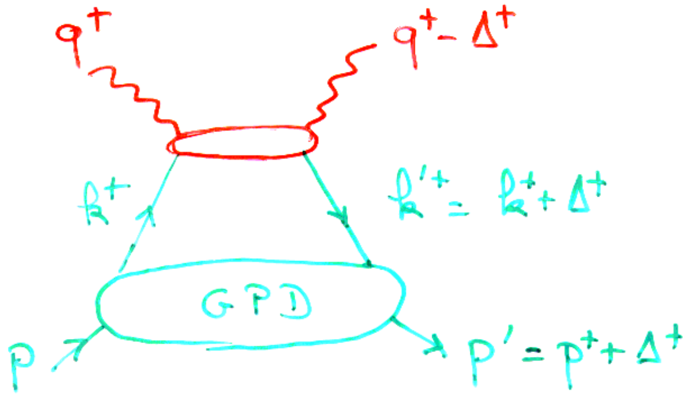
"may" allow to map the spatial distribution of a parton with given x

(role of ζ not yet clear!)

③ Ji 's sum rule

$$\boxed{\int dx \left(h_i + e_i \right) x \;=\; J_i} \qquad i = \text{parton type.}$$

Parton interpretation

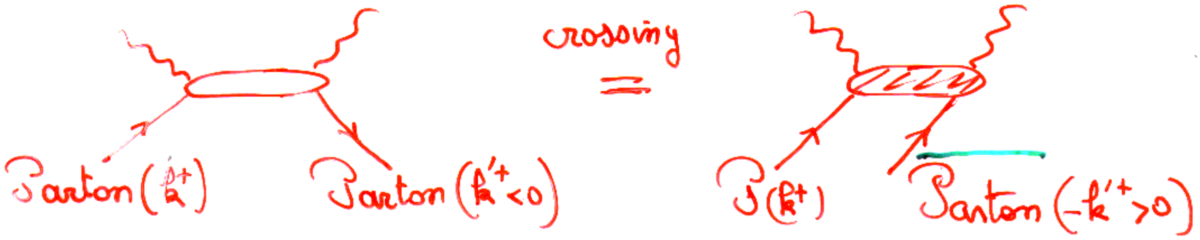


naive parton $\quad p^+ \geq k^+ \geq 0$

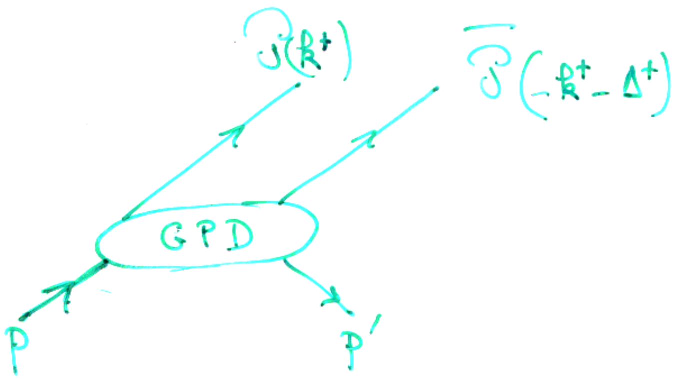
$$p^+ + \Delta^+ \geq k^+ + \Delta^+ \geq 0 \quad \Rightarrow \quad k^+ \geq -\Delta^+$$

$$\boxed{\zeta \leq x \leq 1}$$

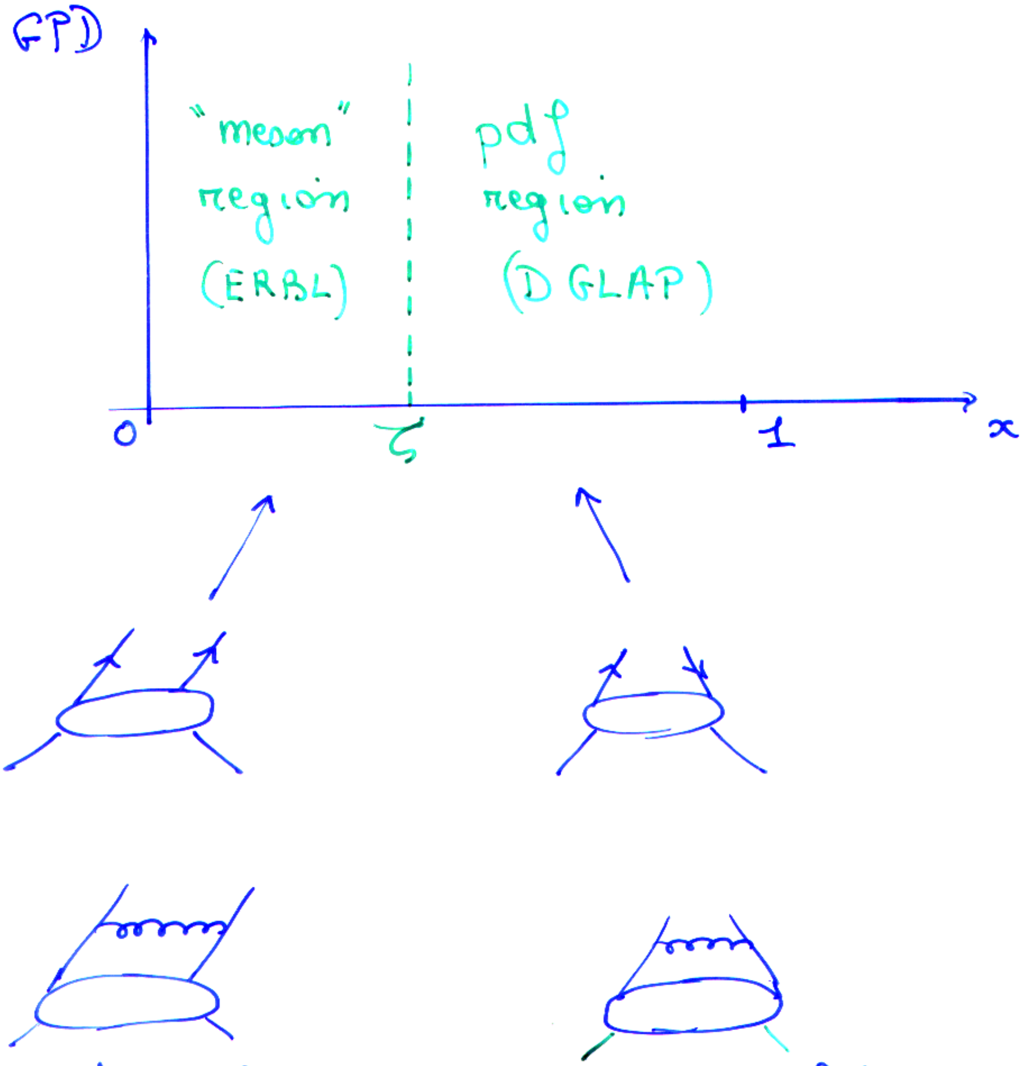
What if $k^+ \leq -\Delta^+$? Then $k'^+ = k^+ + \Delta^+ \leq 0$



So $\quad 0 \leq x \leq \zeta \quad$ corresponds to



" meson like " configuration



evolution follows
ERBL equations

evolution follows
DGLAP equations.

- Evolution and radiative corrections to H and have been worked out NLO

- Explicit calculations exist (Freund McDermott 2001)

Double distributions

remember $T^{VCS} \sim \int dx \frac{1}{x - x_B} GPD(x, \xi, t)$: convolution

To get the GPD's from the data one needs a parametrization which embodies naturally the 2 regimes

$1 > x > \xi$ pdf like

$\xi > x > 0$ meson like

The idea (Radyushkin 1997) is to consider p^+ and Δ^+ as independent

Instead of

$$\bar{\psi}(0)\gamma^+\psi(0, y^-, 0_\perp) = \int dx\, e^{i(xp^+)y^-} GPD(x, \xi, t)$$

define

$$\bar{\psi}(0)\gamma^+\psi(0, y^-, 0_\perp) = \int dX\, dY\, e^{i(Xp^+ + Y\Delta^+)y^-} DD(X, Y, t)$$

$X \longrightarrow$ pdf like behavior

$Y \longrightarrow$ meson like "

by kinematics $\Delta^+ = -\xi p^+ \rightarrow x = X - \xi Y$

$$GPD(x, \xi, t) = \int dX\, dY\, \delta[x - (X - \xi Y)]\, DD(X, Y, t)$$

advantage: X, Y do not depend on ξ

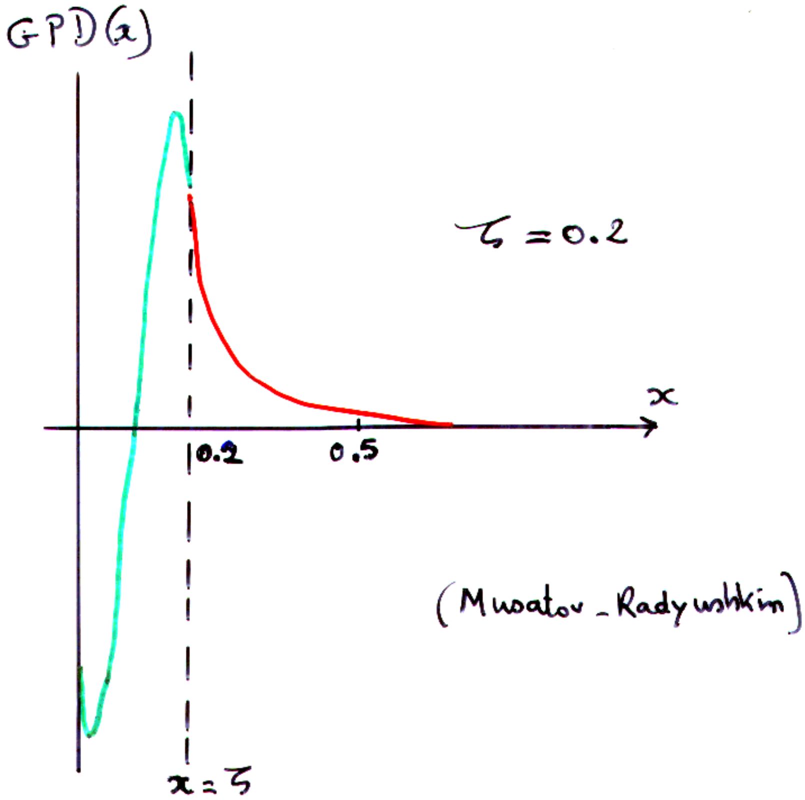
Rule of use for the DD

- make a guess for $DD(x, y, t)$
- deduce the corresponding $GPD(x, \xi, t)$
- compute the DVCS amplitude.

Popular guess

$$DD(x, y, t) = \underbrace{f(x)}_{\text{pdf}} \; \underbrace{\Phi_x(y)}_{\text{meson wave function}} \; \underbrace{g(t)}_{\text{form factor.}}$$

advantage : few parameters.


GPD(x)
ζ = 0.2
x
0.2
0.5
x = ζ
(Musatov - Radyushkin)

The guess

$$DD(x, y, t) = f(x)\, \Phi_x(y)\, g(t)$$

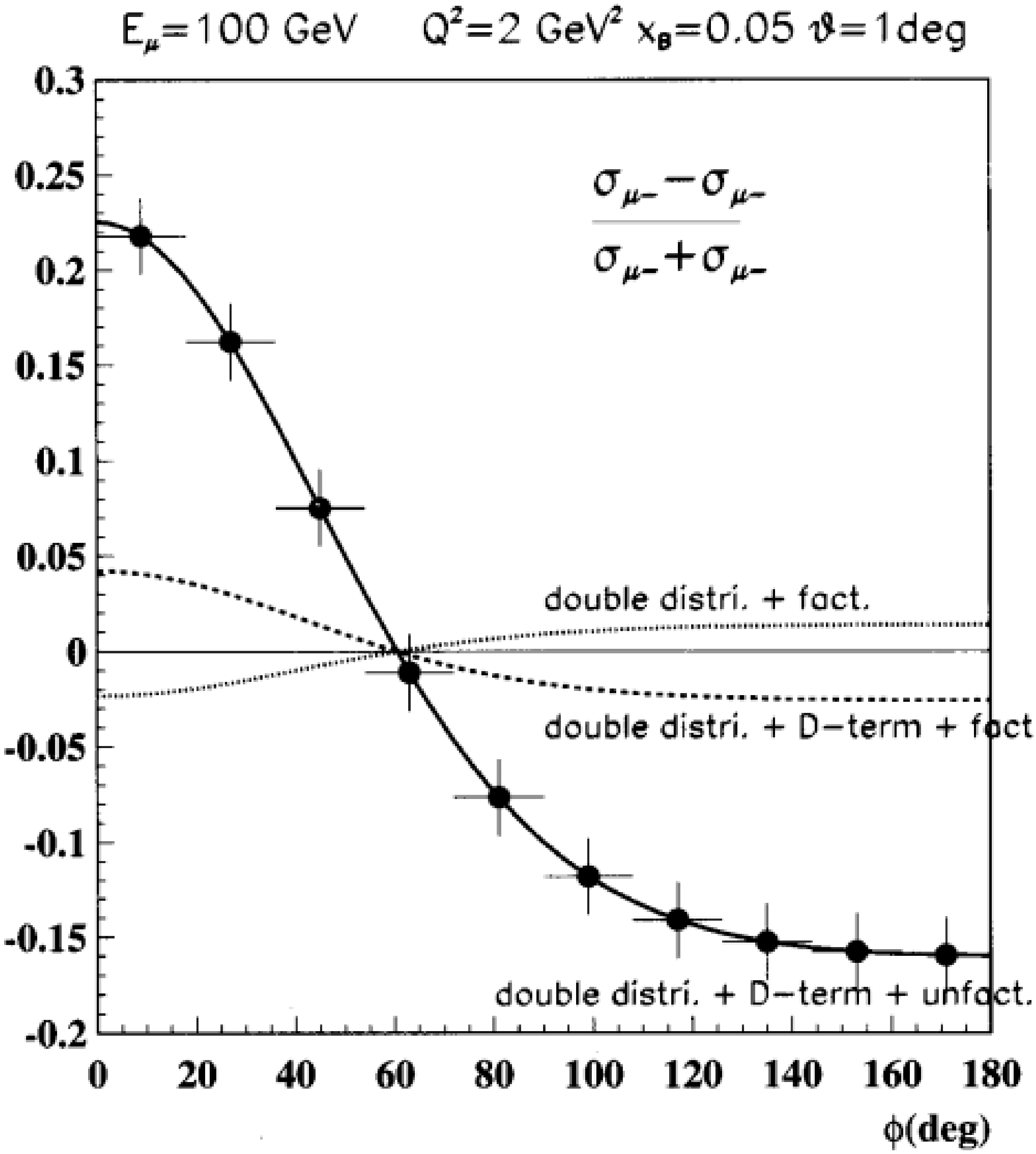
needs improvements:

x distribution cannot be totally independent of t because at small x the size increases

$\longrightarrow$ t dependance steeper at small x

Assuming $\langle R_\perp^2 \rangle \sim \ln \frac{1}{x}$ then

$$f(x)\, \Phi(y)\, g(t) \longrightarrow f(x)\, \Phi(y)\, g\left(t \ln \frac{x_0}{x}\right)$$

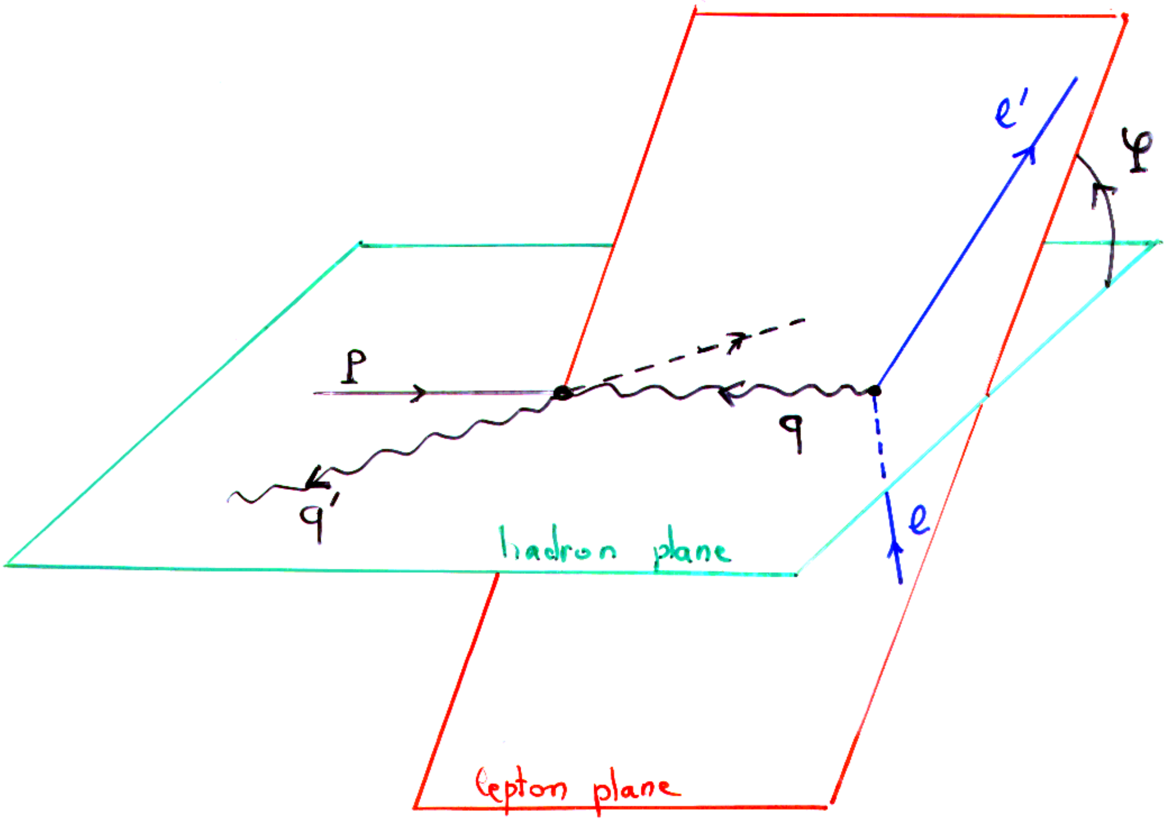


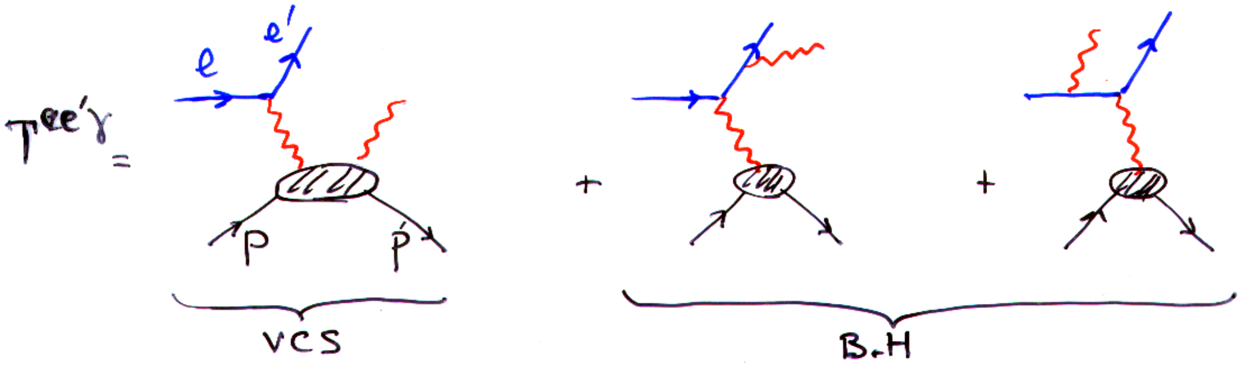
E_μ=100 GeV Q^2=2 GeV^2 x_B=0.05 ϑ=1deg
$\frac{\sigma_{\mu-}-\sigma_{\mu-}}{\sigma_{\mu-}+\sigma_{\mu-}}$
double distri. + fact.
double distri. + D−term + fact
double distri. + D−term + unfact.
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
20
40
60
80
100
120
140
160
180
φ(deg)

Observables and results

- Freund - Mc Dermott Hep-ph/01 11 472
- Belitsky - Müller - Kirchner Hep-ph/01 12 108

what follows is a personal selection ...





$$\sigma^{ee'\gamma} \sim |T^{VCS}|^2 + |T^{BH}|^2 \pm 2\,\mathrm{Re}\left(T^{VCS}\, T^{BH\,*}\right)$$

lepton charge

Concentrate on

$$\Delta\sigma = \sigma(+) - \sigma(-) = 4\,\mathrm{Re}\; T^{DVCS}\; T^{BH\,*}$$

(beam charge difference)

$$\Delta\sigma = (\text{know kinematical factors}) \times \ldots$$

$$\sum_{n=0,3} C_n \cos n\phi + S_n \sin n\phi \quad (+\text{ twist } 4)$$

$n = 0, 1$ do not depend on twist 3 amp.

→ $\boxed{\text{Azymuthal angular distribution allows to project the twist 2 from the data}}$

(up to $\frac{1}{Q^2}$ corrections)

exemple t and x_B small, unpol. target

$$C_1 = F_1(t)\, \mathcal{R}e \int dx \left(\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi+i\epsilon} \right) H(x,\xi,t)$$

$S_1 =$ " $\mathcal{I}m$ "

Charge difference azym distribution

+ polarized (↑ and →) target allows to extract all relevant twist 2 quantities

$$\int dx \left(\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi+i\epsilon} \right) (H \text{ or } E)$$

$$\int dx \left(\frac{1}{x-\xi+i\epsilon} - \frac{1}{x-\xi+i\epsilon} \right) (\tilde{H} \text{ or } \tilde{E})$$

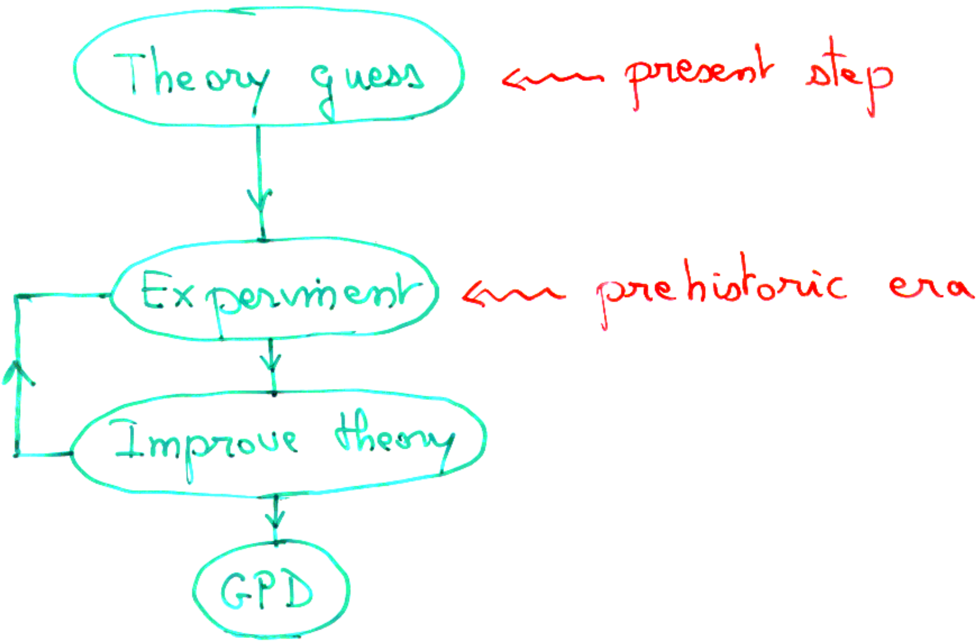
Assuming

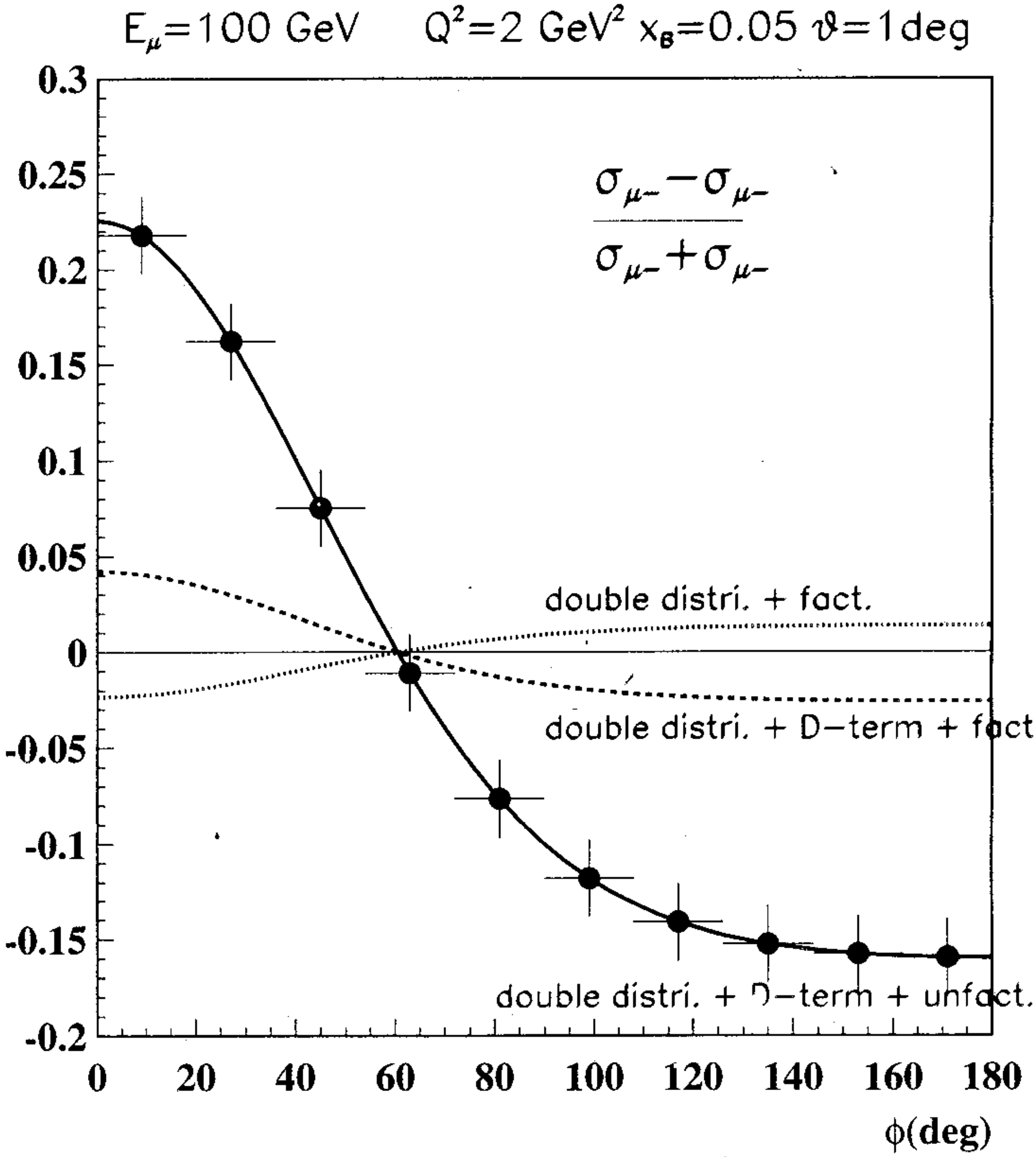
1) a reasonnable parametrization of the double distrib.
2) Q^2 evolution is under control

the only question is :

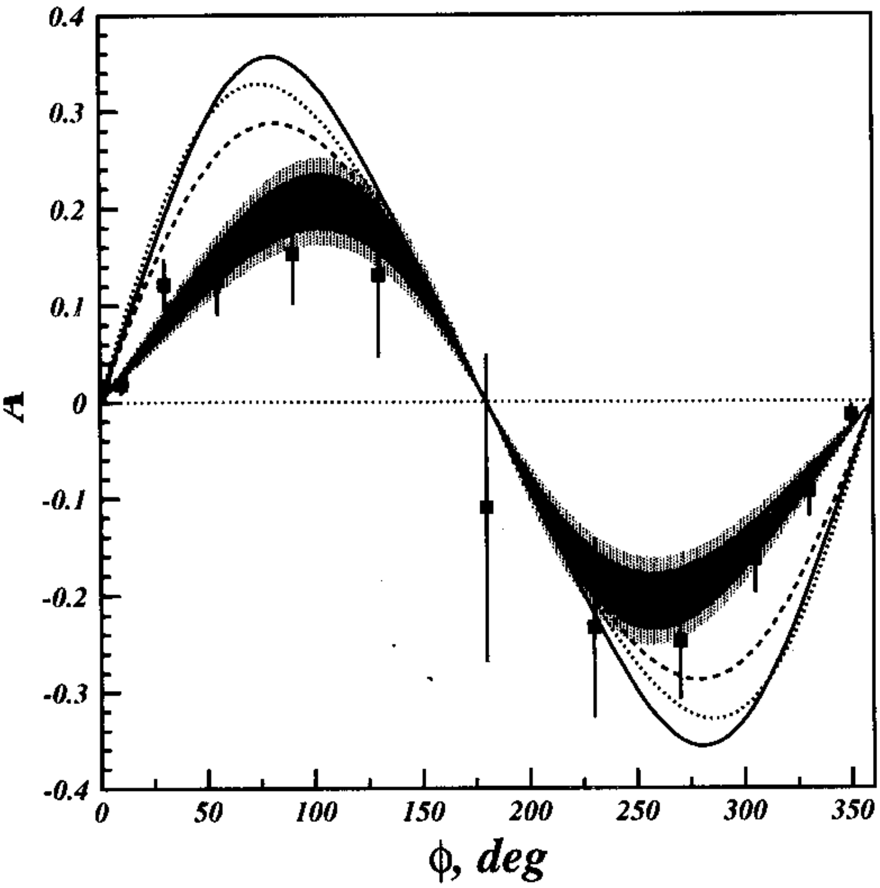
Are the observables sensitive enough to all the input parameters so as to extract the GPD's ?

the answer can only be iterative



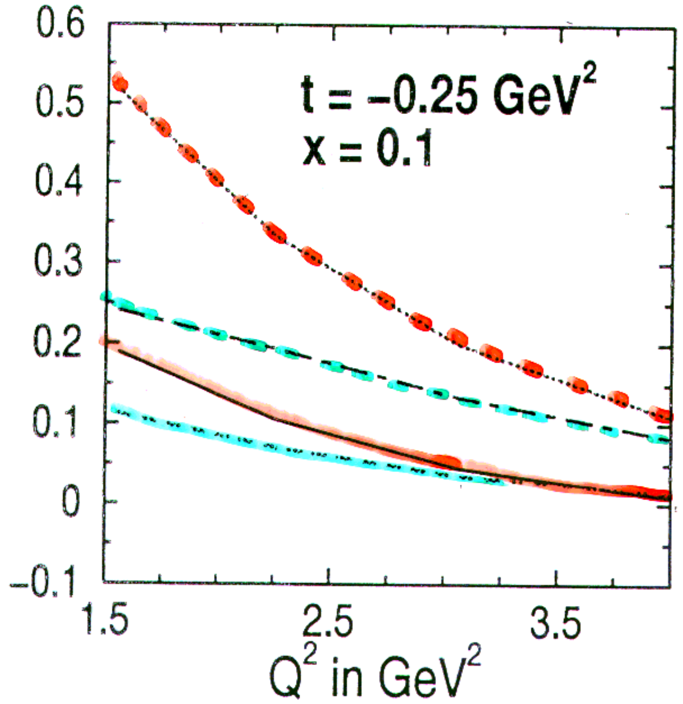


E_μ=100 GeV Q^2=2 GeV^2 x_B=0.05 ϑ=1deg
$\frac{\sigma_{\mu-} - \sigma_{\mu-}}{\sigma_{\mu-} + \sigma_{\mu-}}$
double distri. + fact.
double distri. + D−term + fact
double distri. + D−term + unfact.
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
20
40
60
80
100
120
140
160
180
ϕ(deg)



0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
A
0
50
100
150
200
250
300
350
ϕ, deg

Charge asymmetry (Freund - Mc Dermott 2001)

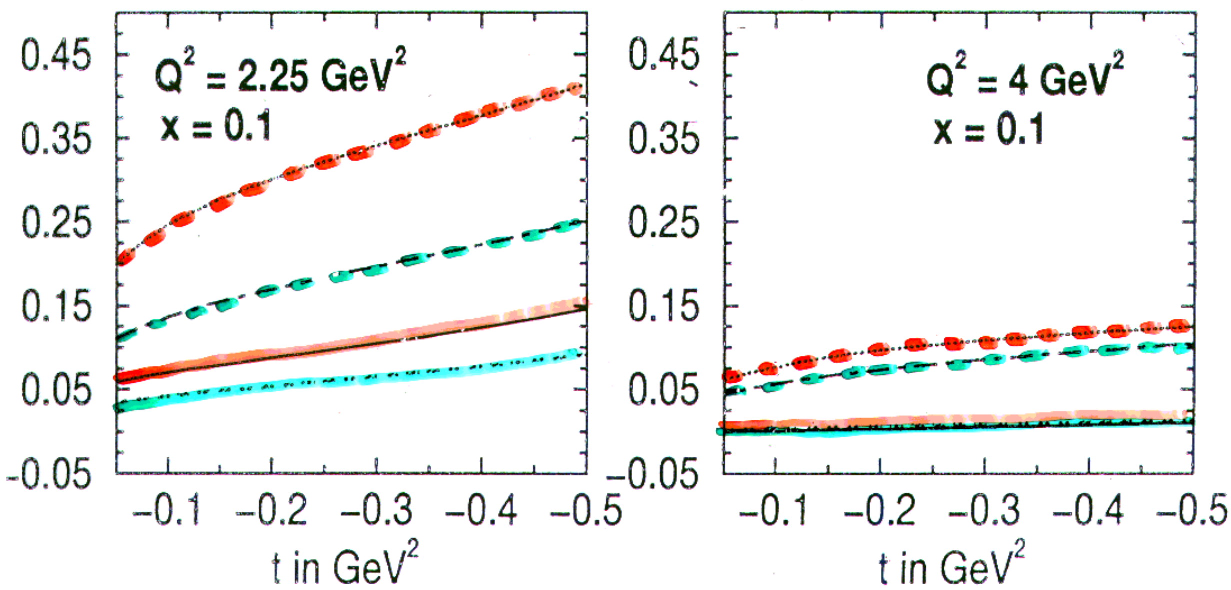


● MRSA

● GRV 98

LO : full line

NLO : dotted line



Prospective for COMPASS (and others)

- Measure beam charge difference
- Have enough angular resolution to project out the twist 2
- Have a sufficient range in Q^2 to allow a control of Q^2 evolution
- Consider the case of polar. targets (to separate $H, E, \tilde{H}, \tilde{E}$)
- Consider meson production to separate flavours
- Compute Ji sum rule
- Do not forget the t dependance (spatial distribution of a parton which carries x)