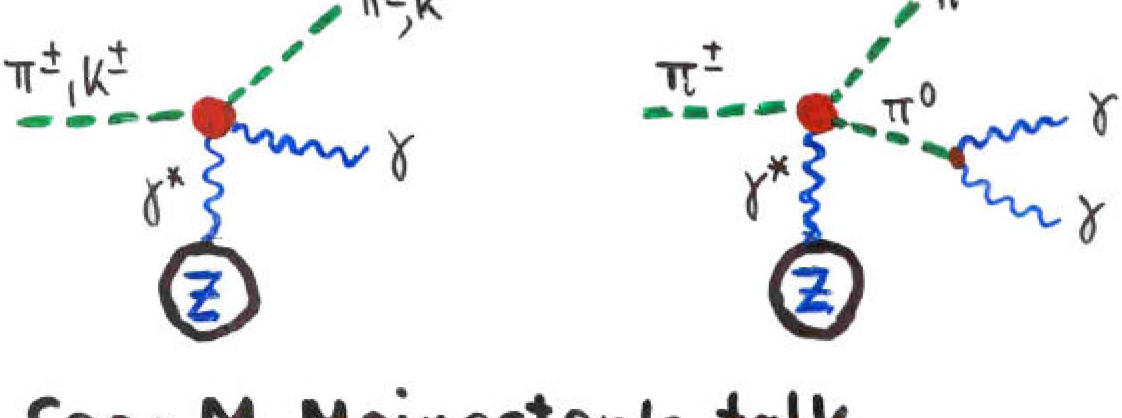
TESTS OF CHIRAL PERTURBATION THEORY WITH COMPASS

(N. Kaiser, TU-München) Trieste, 18.2.2002

- Pion and Kaon electromagnetic polarizabilities: $\pi^\pm\gamma \to \pi^\pm\gamma$, $K^\pm\gamma \to K^\pm\gamma$
- Chiral anomaly of QCD
  $\pi^0 \to 2\gamma$, $\pi^\pm\gamma \to \pi^\pm\pi^0$

Method: Primakoff scattering in Coulombfield of heavy nucleus



See: M. Moinester's talk

What is Chiral Perturbation Theory (CHPT)?

• Symmetries of Quantum Chromodynamics

$$\boxed{\mathcal{L}_{QCD} = \bar{q}\gamma_\mu(i\partial^\mu - g_s G^\mu)q - \frac{1}{4}G_{\mu\nu}^2 - \bar{q}\,\mathcal{M}\,q}$$

quark masses small

Gasser-Leutwyler: $m_u \approx 5\,\text{MeV}$, $m_d \approx 9\,\text{MeV}$, $m_s \approx 175\,\text{MeV}$

$\mathcal{M} \to 0$

$$\boxed{\begin{array}{c}\text{Chiral Symmetry: } SU(3)_L \times SU(3)_R \\ q \to g_L \frac{1}{2}(1-\gamma_5)q + g_R \frac{1}{2}(1+\gamma_5)q\end{array}}$$

• Ground state: Spontaneous chiral symmetry breaking

$$SU(3)_L \times SU(3)_R \to SU(3)_V \text{ flavor sym.}$$

quark condensate: $\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{q}_L q_R + \bar{q}_R q_L|0\rangle \neq 0$

GOLDSTONE THEOREM:

• 8 pseudoscalar Goldstone bosons $\phi = (\pi, K, \eta)$

$$\boxed{\langle \phi^a(q)|A_\mu^b|0\rangle = i\delta^{ab} F_\pi q_\mu}$$

scale parameter: $F_\pi = 92.4\,\text{MeV}$

$m_\pi^2 \sim (m_u + m_d)$, $m_K^2 \sim (m_u + m_s)$, $m_\eta^2 \sim \frac{1}{3}(m_u + m_d + 4m_s)$

• Interaction of Goldstone bosons weak at low energies: $a(\pi\pi) \sim m_\pi^2$

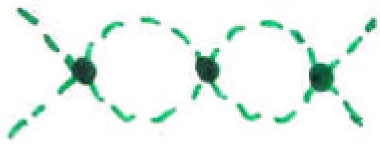
• Small Expansion parameters

$\boxed{m_q/\Lambda\,,\; E/\Lambda}$ $\quad \Lambda \approx 4\pi F_\pi \sim 1\,\text{GeV}$

• Concept of effective field theory

$$\boxed{\mathcal{L}_{eff} = \mathcal{L}[\underbrace{\partial_\mu U(\phi)}_{\text{Goldstone bosons}}, \underbrace{\mathcal{M}}_{\text{quark mass}}, \underbrace{N}_{\text{matter field, nucleon}}, \ldots]}$$

- Uniquely determined by chiral symmetry
- Perturbation Theory ($\hat{=}$ Loop Expansion) with $\mathcal{L}_{eff}$: **Systematic low energy expansion**



L loops

Amplitude $\sim \left(\frac{E}{\Lambda}\right)^{2L+2}$

(Weinberg, Gasser, Leutwyler)

- Perturbative Renormalization through counter terms of higher chiral order

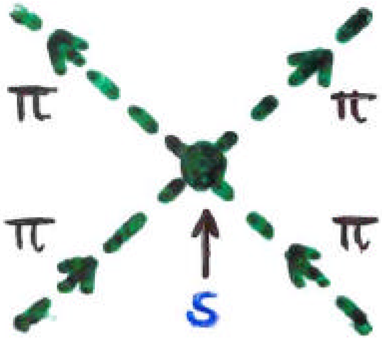
Finite part of low energy constants $\hat{=}$ Resonance exchange $R = (\rho, \omega, \Delta, \ldots)$ Dynamics at higher energy scales

$$\boxed{\textbf{Theory} \longleftrightarrow \textbf{Experiment}}$$

Precise low energy data:

Test of chiral symmetry, as well as its spontaneous and explicit breaking

PION-PION SCATTERING



$$A = \frac{s - m_\pi^2}{f_\pi^2}$$ (Weinberg '66)

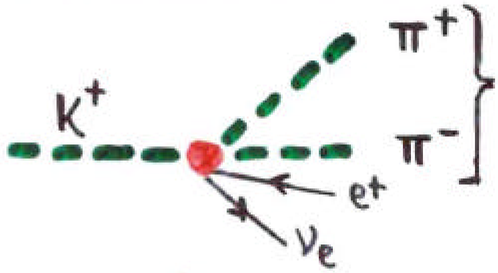
- $\sim E^2$, $\pi\pi$-interaction <u>weak</u> at low energies

• Isospin-0 S-wave scattering length

$$\boxed{a_0^0 = \frac{7 m_\pi^2}{32\pi f_\pi^2}\Big[1 + \overset{\text{1-loop}}{9L} + \overset{\text{2-loop}}{81.6L^2} + \dots\Big], \quad L = -\left(\frac{m_\pi}{4\pi f_\pi}\right)^2 \ln\frac{m_\pi}{\Lambda} \\ = 0.16 + 0.04 + 0.02 = 0.22 \pm 0.01}$$

Colangelo et al.

• Experimental determination of a_0^0



Phase shift $\delta_0^0 - \delta_1^1$

final state interaction

CERN-Expt ('77)

$$\boxed{a_0^0 = 0.26 \pm 0.05}$$

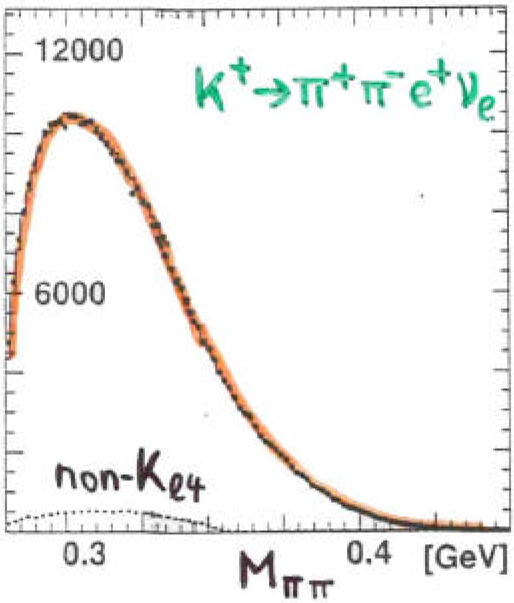
NEW!

- E865 @Brookhaven $\boxed{a_0^0 = 0.216 \pm 0.015}$

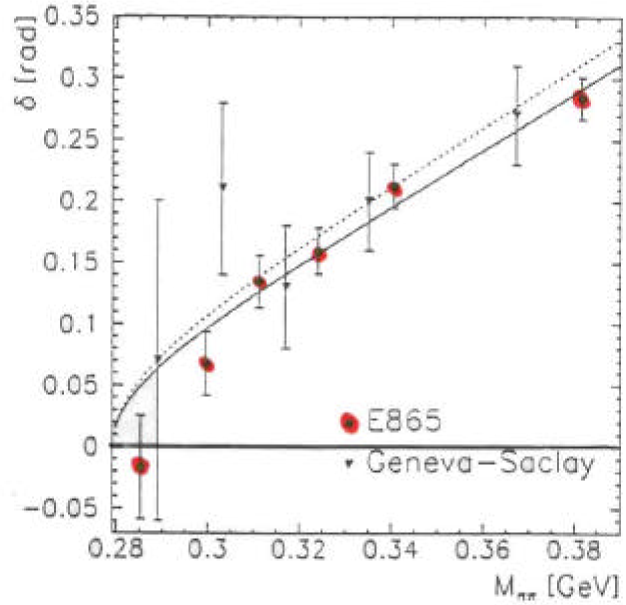
  PRL 87, 221801

- KLOE @Frascati $K^+ \to \pi^+\pi^- e^+\nu_e$

$\pi\pi$ invariant mass distribution



$\pi\pi$ phase shift



- Confirmation of $a_0^0 = 0.22$ implies

$$\boxed{\text{LARGE QUARK CONDENSATE}}$$

$$-\langle 0|\bar{q}q|0\rangle / f_\pi^2 \simeq 1.3\,\text{GeV}$$

- Quark mass expansion of m_π^2 dominated by linear term

$$m_\pi^2 f_\pi^2 = \underbrace{-\langle 0|\bar{q}q|0\rangle\, m_q}_{\text{at least } 94\%} + \mathcal{O}(m_q^2 \ln m_q)$$

Leutwyler et al. PRL 86, 5008 ('01)

- DIRAC @ CERN: Pionium life time (10% accurate)

$$\Gamma\big((\pi^+\pi^-)_{\text{Atom}} \to \pi^0\pi^0\big) = \frac{2}{9}\alpha^3 p^* (a_0^0 - a_0^2)^2 + \dots$$

ChPT prediction: $\tau = (2.9 \pm 0.1) \cdot 10^{-15}$ sec !

Gasser et al. PRD 64, 016008

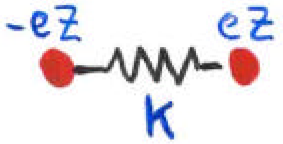
Introduction

- Electric (α) and magnetic (β) polarizabilities describe response to external electric and magnetic fields

Classical: induced dipole moments

$$\vec{d} = \alpha \vec{E}, \qquad \vec{\mu} = \beta \vec{B}$$

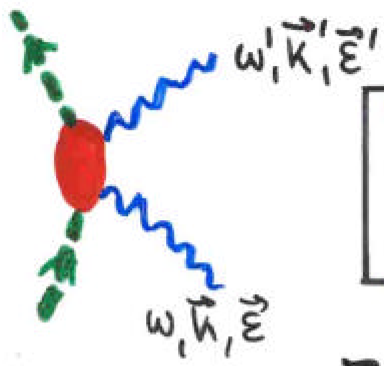
α, β measure of stiffness or rigidity of system



$$d = eZ \cdot 2\ell, \qquad K\ell = ZeE$$

$$\boxed{\alpha = \frac{2(eZ)^2}{K}}$$

Quantum mechanical: Compton-scattering amplitude
low energy-expansion



$$\boxed{T(\gamma\pi \to \gamma\pi) = T_0 + \bar{\alpha}\, \omega\omega'\, \vec{\varepsilon}\,' \cdot \vec{\varepsilon} + \bar{\beta}\, (\vec{\varepsilon}\,' \times \vec{k}') \cdot (\vec{\varepsilon} \times \vec{k}) + \ldots}$$

Thomson amplitude: classical limit

$$\boxed{T_0 = -\frac{e^2 Z^2}{4\pi m}\, \vec{\varepsilon}\,' \cdot \vec{\varepsilon}}$$

Z: electric charge
m: mass of particle

$\leftrightarrow$ Current conservation $\partial^\mu J_\mu^{elm} = 0$

- Forward dispersion relation + optical theorem

$$\boxed{\bar{\alpha}+\bar{\beta} = \frac{1}{2\pi^2}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega^2}\,\sigma_{abs}^{\gamma}(\omega)}$$ Baldin's sum rule

- $(\bar{\alpha}+\bar{\beta})_{\pi^\pm} = (0.39 \pm 0.04?)\cdot 10^{-4}\,\mathrm{fm}^3$

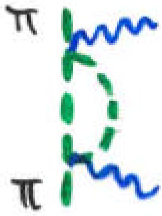
  Petrunkin ('81): Resonances $R^* \to \pi\gamma$

- For comparison: Nucleons

  $(\bar{\alpha}+\bar{\beta})_p = (13.69 \pm 0.14)\cdot 10^{-4}\,\mathrm{fm}^3$ Babusci et al. ('98)

  $(\bar{\alpha}+\beta)_n = (14.40 \pm 0.66)\cdot 10^{-4}\,\mathrm{fm}^3$ photoprod. data

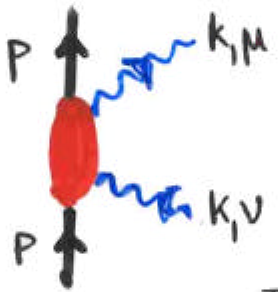
low energy
πN-states



strong suppression
→ chiral anomaly

high mass resonances
$\rho, \cancel{\omega}, a_1 \dots$

- Quantum field theory formalism



Compton-tensor in forward direction

$s = (p+k)^2, \quad p^2 = m^2, \quad k^2 = 0$

$$\boxed{\Theta_{\mu\nu} = (g_{\mu\nu} + \dots)\,A(s) + k_\mu k_\nu\,B(s)}$$

Def.:

$$\bar{\alpha}+\bar{\beta} = -\frac{m}{4\pi}A''(m^2), \quad \bar{\beta} = -\frac{1}{8\pi m}B(m^2)$$

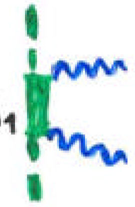
at threshold

$$h_A^{exp.} = (8.7 \pm 1.2)\cdot 10^{-5}\,\mathrm{MeV}^{-1} \quad (\mathrm{PDG})$$

$$\boxed{(\bar{\alpha}-\bar{\beta})_{\pi^\pm} = \frac{e^2 h_A}{2\pi\sqrt{2}\, m_\pi f_\pi^2} = (5.4 \pm 0.8)\cdot 10^{-4}\,\mathrm{fm}^3}$$

- Low energy constant: $L_9^r + L_{10}^r \simeq 1.4 \cdot 10^{-3}$

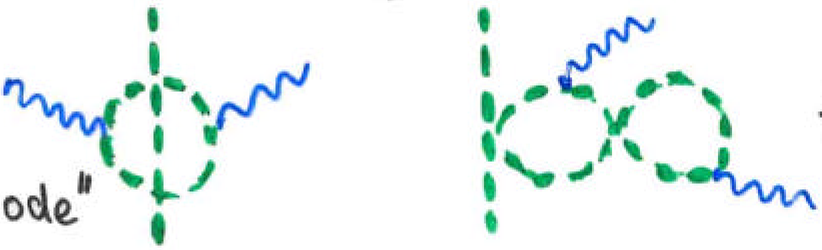
  saturated by $\rho(770)$ and $a_1(1260)$-exchange

ρ, a_1 

$$\underset{\uparrow}{L_9^r} + L_{10}^r \simeq [\underset{\rho}{(6.9-10)} + \underset{a_1}{4.5}]\cdot 10^{-3} \quad (\text{Ecker et al.})$$

pion charge radius $\langle r^2 \rangle_\pi = (0.43 \pm 0.02)\,\mathrm{fm}^2$

- $\mathcal{O}(p^6)$: 2-loop graphs + ...

"acnode"  + ...

<u>Neutral pions:</u>

$$\boxed{\begin{aligned}(\bar{\alpha}+\bar{\beta})_{\pi^0} &= (1.2 \pm 0.3)\cdot 10^{-4}\,\mathrm{fm}^3 \\ (\bar{\alpha}-\bar{\beta})_{\pi^0} &= (-1.9 \pm 0.2)\cdot 10^{-4}\,\mathrm{fm}^3\end{aligned}}$$

Belluci et al. NPB 423 ('94) 80

Quark models → larger values

<u>Charged pions:</u>

$$\boxed{\begin{aligned}(\bar{\alpha}+\bar{\beta})_{\pi^\pm} &= (0.3 \pm 0.1)\cdot 10^{-4}\,\mathrm{fm}^3 \\ (\bar{\alpha}-\bar{\beta})_{\pi^\pm} &= (4.4 \pm 1.0)\cdot 10^{-4}\,\mathrm{fm}^3\end{aligned}}$$

U. Bürgi, NPB 479 ('96) 392

error estimate: uncertainties of higher order couplings

7

$h_A^{exp.} = (8.7 \pm 1.2) \cdot 10^{-5}\,\mathrm{MeV}^{-1}$ (PDG)

$$\boxed{(\bar{\alpha} - \bar{\beta})_{\pi^\pm} = \frac{e^2 h_A}{2\pi\sqrt{2}\, m_\pi f_\pi^2} = (5.4 \pm 0.8) \cdot 10^{-4}\,\mathrm{fm}^3}$$

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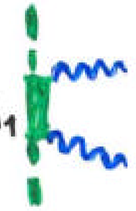
ρ, a_1 

$$\underset{\uparrow}{L_9^r} + L_{10}^r \simeq [\underbrace{(6.9 - 10)}_{\rho} + \underbrace{4.5}_{a_1}] \cdot 10^{-3} \quad \text{(Ecker et al.)}$$

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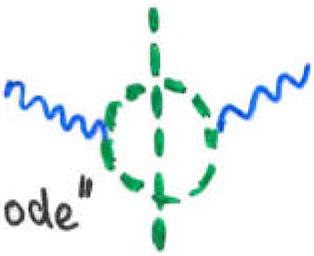
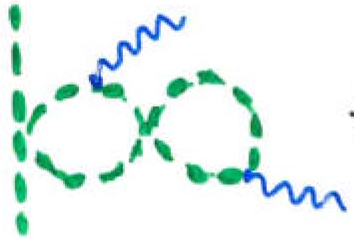
"acnode"   + ...

<u>Neutral pions:</u>

$$\boxed{\begin{aligned} (\bar{\alpha} + \bar{\beta})_{\pi^0} &= (1.2 \pm 0.3) \cdot 10^{-4}\,\mathrm{fm}^3 \\ (\bar{\alpha} - \bar{\beta})_{\pi^0} &= (-1.9 \pm 0.2) \cdot 10^{-4}\,\mathrm{fm}^3 \end{aligned}}$$

Belluci et al. NPB 423 ('94) 80

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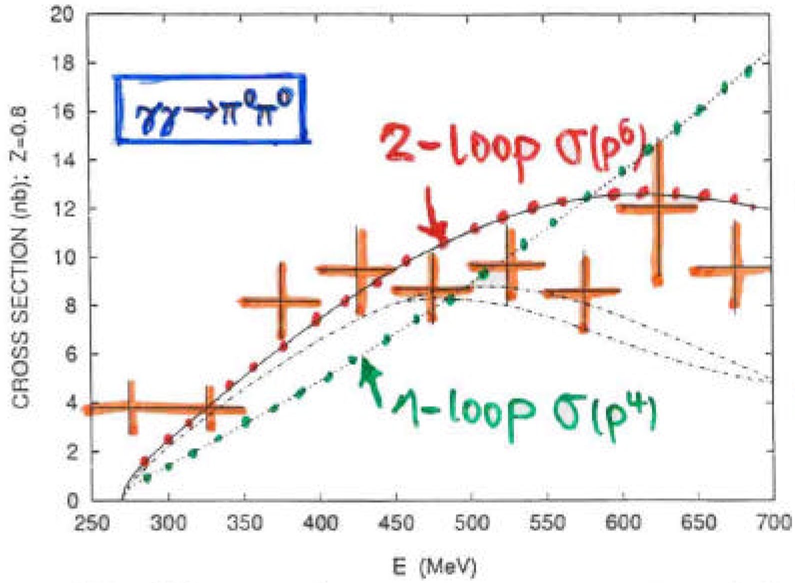
error estimate: uncertainties of higher order couplings

Cross sections: Pion-pair production by photons

Neutral pions:

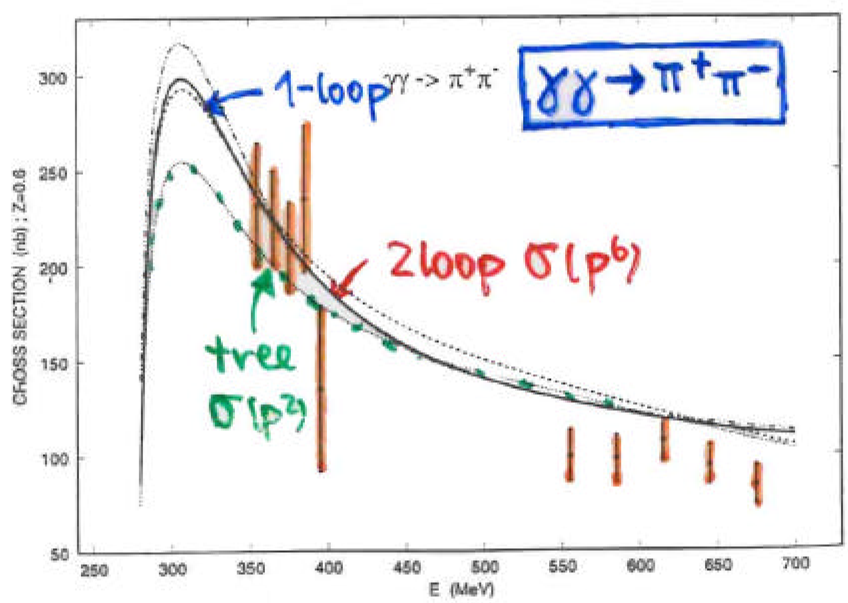


Belluci et al. NPB423('94)80

Data: Crystal Ball collaboration

- tree $\sigma(p^2)$ vanishes, 1-loop $\sigma(p^4)$ below data
- 2-loop $\sigma(p^6)$ in good agreement with data

Charged pions:



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Data: Mark II collaboration

Existing empirical determinations

i) Primakoff scattering: $\pi^- Z \to \pi^- \gamma Z$

- analyzing with constraint: $(\bar{\alpha}+\bar{\beta})_{\pi^\pm} = 0$

$$(\bar{\alpha}-\bar{\beta})_{\pi^\pm} = \begin{cases} 40 \pm 24 & \text{(Lebedev, } \gamma p \to \gamma \pi^+ n\text{)} \\ 13.6 \pm 2.8 & \text{(Serpukhov)} \end{cases}$$

- relaxing $(\bar{\alpha}+\bar{\beta})_{\pi^\pm} = 0$ constraint: Serpukhov data $\to$

$$(\bar{\alpha}+\bar{\beta})_{\pi^\pm} = 1.4 \pm 4.0 \qquad \text{(Antipov et al.)}$$
$$(\bar{\alpha}-\bar{\beta})_{\pi^\pm} = 15.6 \pm 7.8 \qquad \text{ChPT-pred.} \times 4$$

ii) $\gamma\gamma \to \pi^+\pi^-$ data: unitarized S-wave + adjust $\bar{\alpha}-\bar{\beta}$

$$(\bar{\alpha}-\bar{\beta})_{\pi^\pm} = 4.8 \pm 1.0 \quad \text{(Mark II)}$$

contradicts Serpukhov!

- taking into account D-waves, (Kaloshin et al.)

$$(\bar{\alpha}+\bar{\beta})_{\pi^\pm} = \begin{cases} 0.22 \pm 0.08 & \text{(Mark II)} \\ 0.33 \pm 0.06 & \text{(CELLO)} \end{cases}$$

consistent with CHPT

Conclusion: Using chiral $\gamma\gamma \to \pi^+\pi^-$ amplitude as an interpolation, statistical error in present low energy data and uncertainty in LECs do not allow to pin down $(\bar{\alpha} \pm \bar{\beta})_{\pi^\pm}$ to reasonable accuracy.

iii) $\gamma\gamma \to \pi^0\pi^0$ data (Kaloshin et al.)

$$(\bar\alpha-\bar\beta)_{\pi^0} = -1.1 \pm 1.7 \quad \text{(Crystal Ball)}$$
$$(\bar\alpha+\bar\beta)_{\pi^0} = 1.00 \pm 0.05 ?$$ consistent with CHPT

Threshold amplitude $\gamma\gamma \to \pi^0\pi^0$ is quite insensitive to large changes at Compton threshold

iv) Dispersion sum rules (Petrunkin ...)

$$(\bar\alpha+\bar\beta)_{\pi^0} = 1.04 \pm 0.07$$
$$(\bar\alpha+\bar\beta)_{\pi^\pm} = 0.39 \pm 0.04$$
consistent with CHPT

however:

$$(\bar\alpha-\bar\beta)_{\pi^0} \approx -10, \quad (\bar\alpha-\bar\beta)_{\pi^\pm} \approx 10$$ large!

Kaon polarizabilities

$\mathcal{O}(p^4)$: 1-loop

SU(3)-symmetry

$$\boxed{\begin{aligned}(\bar\alpha+\bar\beta)_{K^\pm} &= 0\\ (\bar\alpha-\bar\beta)_{K^\pm} &= \frac{2e^2}{\pi m_K f_K^2}(L_9^r+L_{10}^r)\\ &= (1.0 \pm 0.2)\cdot 10^{-4}\,\mathrm{fm}^3\end{aligned}}$$

2-loop calc.
Schilcher et al (Mainz)

Need for precise measurements!

Chiral Anomaly of QCD

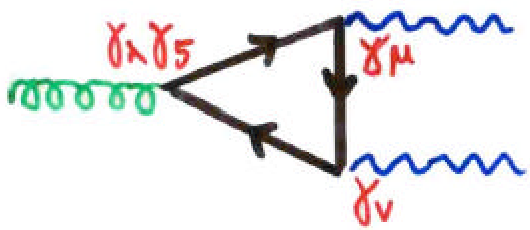
What is an anomaly?

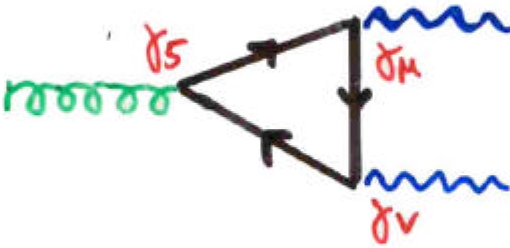
> **C**lassical symmetry of a theory is
> absent at quantum level

Symmetry breaking through quantization

Example: Fermions of mass m couple to photons

$$\left.\begin{aligned}\text{Vector current: } & \partial^\mu(\bar\psi\gamma_\mu\psi)=0\\ \text{axial current: } & \partial^\mu(\bar\psi\gamma_\mu\gamma_5\psi)=2im\bar\psi\gamma_5\psi\end{aligned}\right\}\text{classical}$$





Ward-identities for vector and axial current can not be fulfilled simultaneously

$$\partial^\mu(\bar\psi\gamma_\mu\bar\psi)=0$$
$$\partial^\mu(\bar\psi\gamma_\mu\gamma_5\bar\psi)=2im\bar\psi\gamma_5\psi+\frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

(abelian) anomaly

- Fermionic path integral measure $d\bar\psi\, d\psi$ not invariant under axial transformations
- Deep mathematical origin: Atiyah-Singer Index-Theorems: spectrum of Dirac-operator in gauge field is not left-right symmetric

- Non-abelian gauge anomalies

$$e^{i\Gamma[V,A]} = \int d\bar\psi\, d\psi\; e^{i\int d^4x\, \bar\psi(i\not\partial + \not V + \not A\gamma_5)\psi}$$

$$\delta^a_{\text{vector}}\,\Gamma[V,A] = 0$$

$$\delta^a_{\text{axial}}\,\Gamma[V,A] = \frac{1}{32\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\,\lambda^a\Big\{V_{\mu\nu}V_{\alpha\beta} + \frac{1}{3}A_{\mu\nu}A_{\alpha\beta}$$
$$+ \frac{8i}{3}\left(A_\mu A_\nu V_{\alpha\beta} + A_\mu V_{\nu\alpha}A_\beta + V_{\mu\nu}A_\alpha A_\beta\right) - \frac{32}{3}A_\mu A_\nu A_\alpha A_\beta\Big\}$$

anomaly explicitely known

Effective low energy theory:

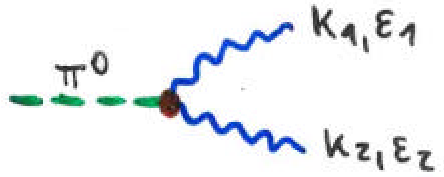
Wess-Zumino-Witten-term

represents chiral anomaly of QCD (with quarks)

$$\boxed{\begin{aligned}\mathcal{L}^{(4)}_{WZW} = &-\frac{ie}{16\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}A_\mu\,\mathrm{tr}\,\overset{\gamma\to 3\pi}{\{\underset{\text{charge operator}}{Q},U^+\}}\,\partial_\nu U U^+\partial_\alpha U U^+\partial_\beta U\\ &-\frac{e^2}{8\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}\,\underset{\pi^0\to 2\gamma}{\partial_\mu A_\nu A_\alpha}\,\mathrm{tr}\Big(Q^2U^+\partial_\beta U + Q^2\partial_\beta U U^+ + \tfrac{1}{2}Q\,\partial_\beta U\, Q U^+\\ &\qquad - \tfrac{1}{2}QUQ\,\partial_\beta U^+\Big) + \ldots\end{aligned}}$$

A_μ = photon field

- Predictions: $\pi^0 \to 2\gamma$

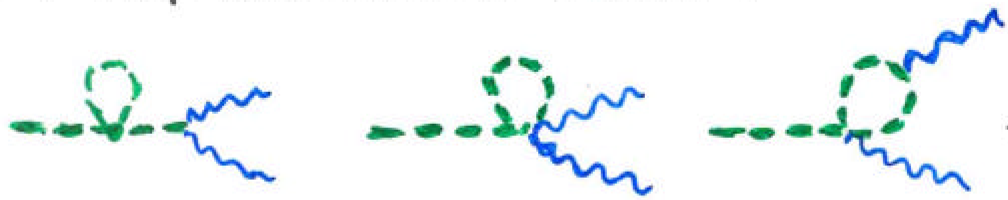


$$T(\pi^0\to 2\gamma) = -\frac{e^2}{4\pi^2 f_\pi}\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon_1^\mu\varepsilon_2^\nu k_1^\alpha k_2^\beta$$

$$\boxed{\Gamma(\pi^0\to 2\gamma) = \frac{\alpha^2 m^3_{\pi^0}}{64\pi^3 f_\pi^2} = 7.73\ \text{eV}}$$

experiment: (7.7 ± 0.7) eV   Williams et al. PRD 38 ('88) 1365

- 1-loop corrections in CHPT



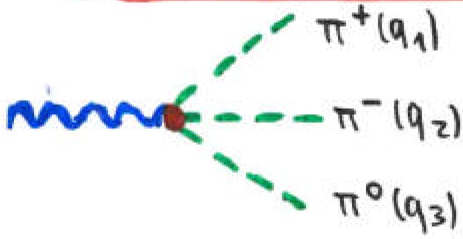
$+\ldots$

Donoghue et al. PRL 55('88)2766

loop corrections $\triangleq$ renormalization of

f_π(chiral limit) $\rightarrow$ f_π(physical)

- **$\gamma \rightarrow \pi^+\pi^-\pi^0$ -Vertex**



$$\underbrace{\frac{e}{4\pi^2 f_\pi^3}}_{F_{3\pi} = 9.72\ \mathrm{GeV}^{-3}} \varepsilon_{\mu\nu\alpha\beta}\, \varepsilon^\mu q_1^\nu q_2^\alpha q_3^\beta$$

Serpukhov experiment: $\pi^- Z \rightarrow \pi^-\pi^0 Z$

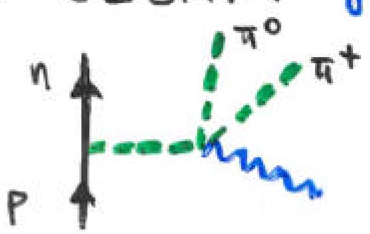
$F_{3\pi}^{(exp)} = (12.9 \pm 1.0)\ \mathrm{GeV}^{-3}$ deviates by 3σ!

- loop corrections Bijnens et al. PLB 237('88)488

$+\ldots$

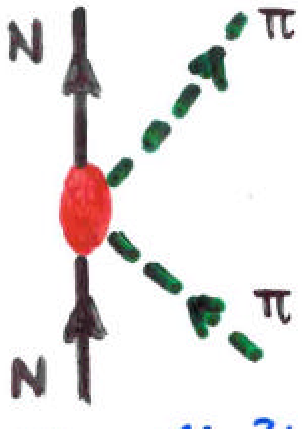
$F_{3\pi}$ increases by ~10%, not const. over phase space

- CEBAF: $\gamma p \rightarrow \pi^+\pi^0 n$ with linearly polarized photons



Chew-Low extrapolation to π-pole $t = m_\pi^2$

S-wave πN-scattering lengths



$$a^- = \frac{1}{3}\left(a_{1/2} - a_{3/2}\right) = \frac{m_\pi}{8\pi f_\pi^2\left(1+\frac{m_\pi}{M_N}\right)} = 7.9$$

$$a^+ = \frac{1}{3}\left(a_{1/2} + 2a_{3/2}\right) = 0 \quad [10^{-2} m_\pi^{-1}]$$

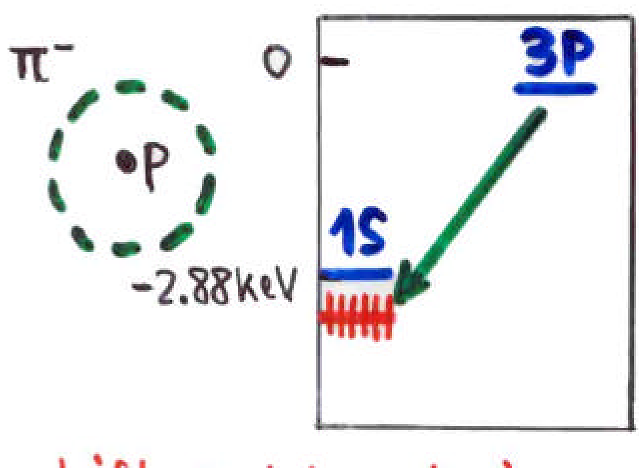
(Tomozawa-Weinberg '66)

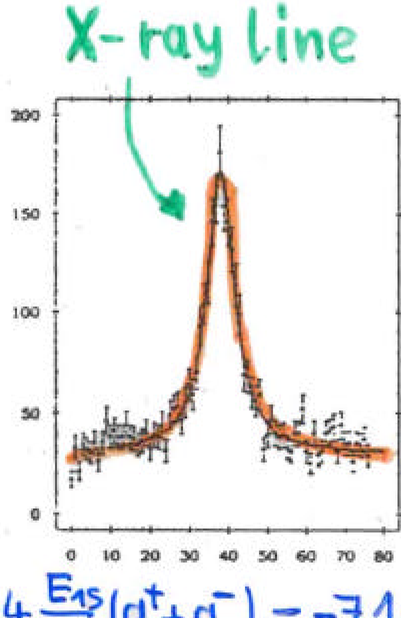
$I_{\pi N} = 1/2, 3/2$

- **Experimental values:** $[10^{-2} m_\pi^{-1}]$

	a^-	a^+
πN-dispersion relation analysis (Höhler, ...)	9.2 ± 0.2	-0.8 ± 0.4
Pionic hydrogen	9.1 ± 0.4	-0.2 ∓ 0.4

PSI-Expt: Schröder et al PLB 469, 25





shift and broadening of 1S-level

$$\varepsilon_{1S} = -4\,\frac{E_{1S}}{r_B}\left(a^+ + a^-\right) = -7.1\,\text{eV}$$

$$\Gamma_{1S} \sim (a^-)^2 \qquad = 0.97\,\text{eV}$$

$$T^-(m_\pi)\big|_{exp.} := 1.85 \pm 0.08 \text{ fm}$$
$$= 4\pi\left(1+\frac{m_\pi}{M_N}\right)a^-$$

Chiral Expansion:

$$T^-(m_\pi) = \frac{m_\pi}{2F_\pi^2}$$

1.592

Current algebra
15 % too small

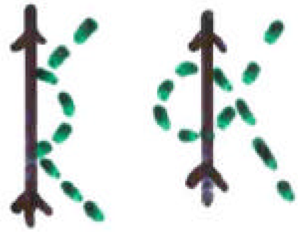
N-pole

$$+ \frac{g_{\pi N}^2 m_\pi^3}{8M^4} + \frac{m_\pi^3}{16\pi^2 F_\pi^4}\left(1 - 2\ln\frac{m_\pi}{\lambda}\right)$$

0.016

0.243 $(\lambda = m_\Delta)$

Loops



"counter terms"

Δ, N^*

$$+ \frac{g_{\pi N}^2 m_\pi^3}{2M^2 M_\Delta^2}\left(Z - \tfrac{1}{2}\right)^2 + \frac{R\, g_{\pi N}^2 m_\pi^3}{8M^2(M+M^*)^2}$$

$0.036\left(Z-\frac{1}{2}\right)^2$ $\qquad$ $0.002R \approx 0$

$-0.8 < Z < 0.3 \leftarrow$ off-shell parameter $\qquad R = 0.25 \ldots 1$

$\approx$ error bar

$+\, \mathcal{O}(m_\pi^5) \leftarrow$ 2-loop (small)

- $\mathcal{O}(m_\pi^2)$ and $\mathcal{O}(m_\pi^4)$ are zero.
- Only <u>Loop correction</u> closes gap between C.A. and exp. value.