QCD Evolution of Transversity in LO and NLO

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* The *Insubri* were a Celtic tribe originally from across the Alps, who in the 5th. century B.C. settled roughly the area now known as Lombardy.

Outline

- History and Notation
- Operator Product Expansion & Co.
- Leading Order QCD Evolution
- Next-to-Leading Order QCD Evolution
- Effects of Evolution on Asymmetries
- Evolution and the Soffer Bound
- A DIS Definition
- DIS–DY K Factor
- Comments and Conclusions

History

- Transversity introduced by Ralston and Soper (1979) in Drell–Yan
- LO Anomalous Dimensions first calculated by Baldracchini *et al*. (1981) . . . and forgotten!
- . . . re-calculated by Artru and Mekhfi (1990)

. . . also unwittingly calculated (in g_2 evolution) by:

- Kodaira *et al*. (1979)
- Bukhvostov, Kuraev and Lipatov (1983)
- Ratcliffe (1986)

History

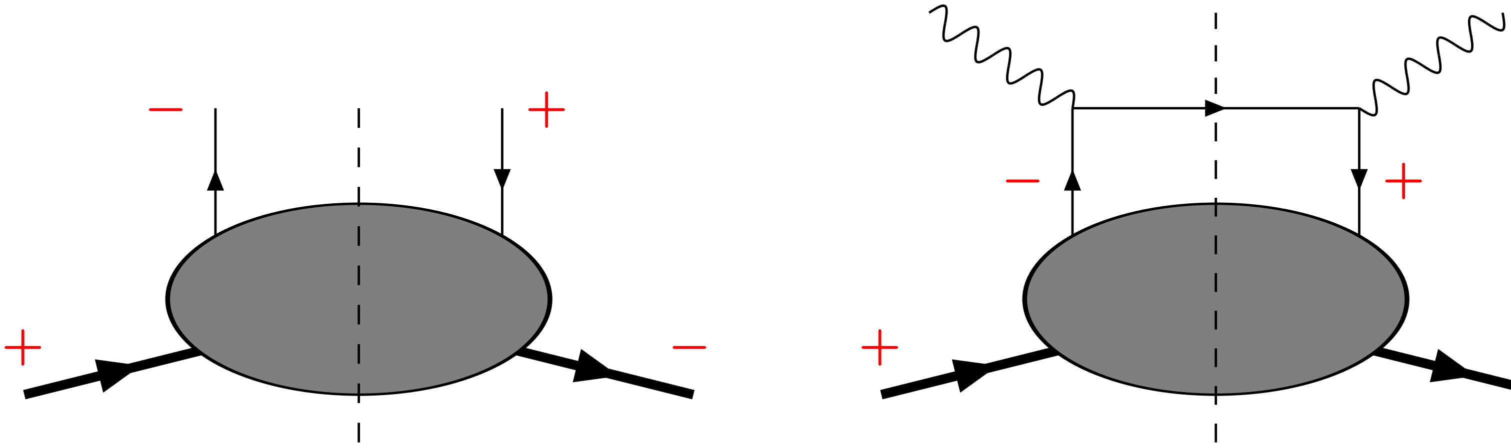
NLO Anomalous Dimensions calculated by:

- Hayashigaki, Kanazawa and Koike (1997)
- Kumano and Miyama (1997)
- Vogelsang (1998)

The effects of evolution have been studied by a number of authors

For more details see, e.g., Barone, Drago and Ratcliffe, *Phys. Rep.* **359** (2002) 1

Chirality Flip



(a) (b)

(a) Chirally-Odd Hadron–Quark Amplitude for h_1

(b) Chirality-Flip Forbidden DIS Handbag Diagram

N.B. Chirality is Not a Problem if the Quarks Connect to Different Hadrons

Basics

Transversity is one of three twist-two structures:

$$f(x) = \int \frac{d\xi^-}{4\pi}\, e^{ixP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle$$

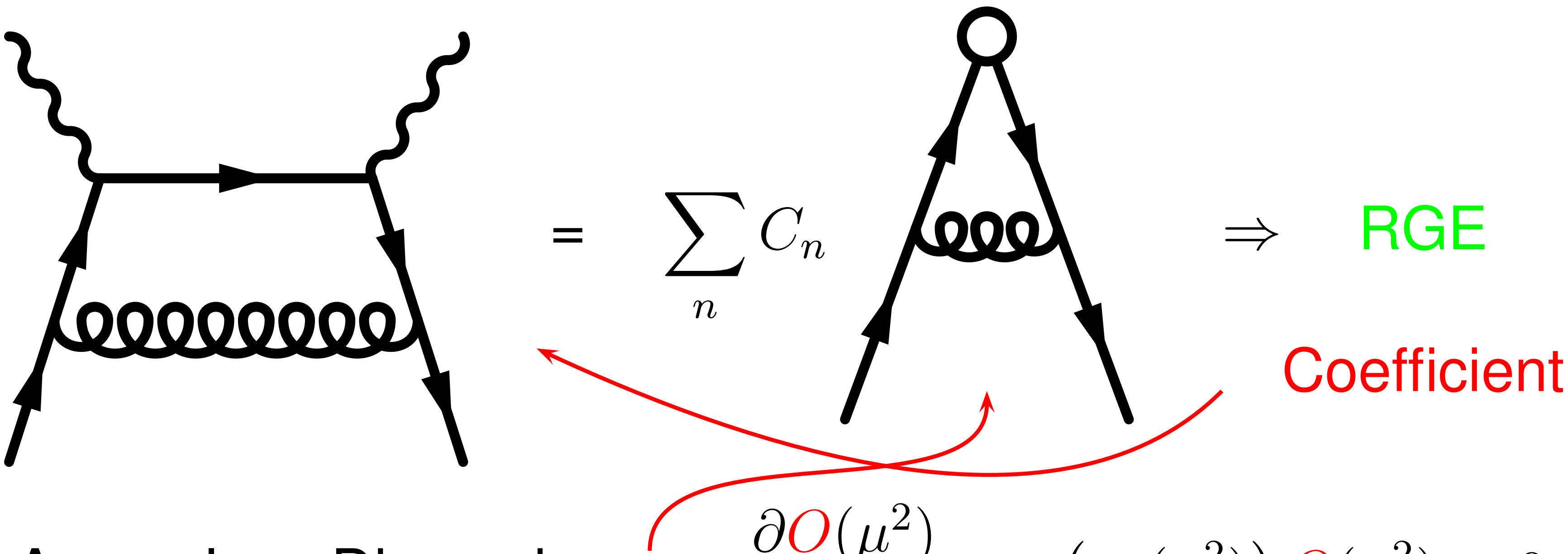
$$\Delta f(x) = \int \frac{d\xi^-}{4\pi}\, e^{ixP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle$$

$$\Delta_T f(x) = \int \frac{d\xi^-}{4\pi}\, e^{ixP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\gamma^1\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle$$

The γ_5 matrix signals spin dependence.

The extra γ^1 matrix in $\Delta_T f(x)$ signals the helicity-flip that precludes transversity contributions in DIS.

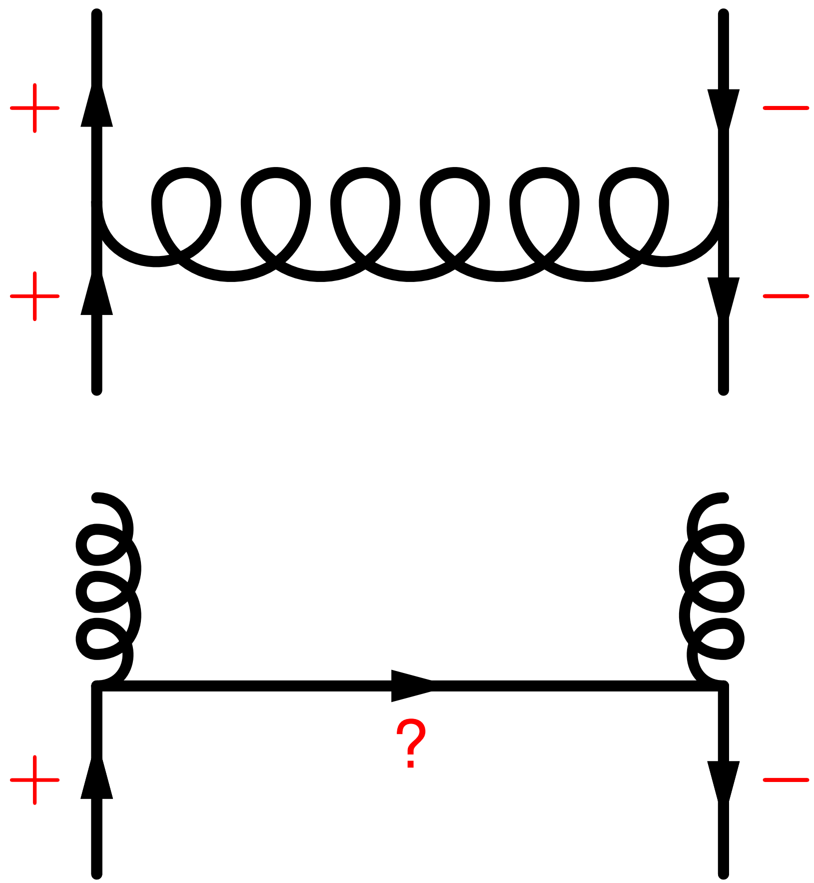
OPE



Anomalous Dimension γ: $\dfrac{\partial O(\mu^2)}{\partial \ln \mu^2} + \gamma\left(\alpha_s(\mu^2)\right) O(\mu^2) = 0$

Solution: $O(Q^2) = O(\mu^2)\, \exp\left[-\int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \mathrm{d}\alpha_s \frac{\gamma(\alpha_s)}{\beta(\alpha_s)}\right]$

Ladder Diagram Summation



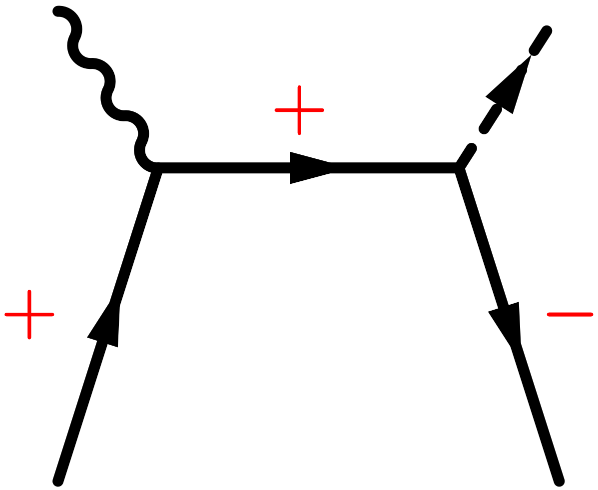
Universal Evolution Kernel

Gluon–Fermion Mixing

Not Allowed

LO QCD evolution of transversity is non-singlet like

Interpolating Currents



$= \quad J_V \cdot J_S$ a method by

Ioffe and Khodjamirian (1995)

A first attempt at calculating γ with this method gave an apparent contradiction – corrected by Blümlein (2001):

While the vector current J_V is conserved and therefore has $\gamma_V = 0$, the scalar current J_S is not conserved and thus has $\gamma_S \neq 0$.

Renormalization Group

The product of two currents may be expanded as

$$J_V(z)\cdot J_S(0)=\sum_n C(n;z)\,O(n;0)$$

The RGE for the Wilson coefficients $C(n;z)$ is

$$\left[\mathcal{D}+\gamma_{J_V}(g)+\gamma_{J_S}(g)-\gamma_O(n;g)\right]C(n;z)=0$$

Thus, the Compton amplitude correction has a coefficient

$$\gamma_C(n;g)=\gamma_{J_V}(g)+\gamma_{J_S}(g)-\gamma_O(n;g)$$

$\left(\mathcal{D}=\mu^2\frac{\partial}{\partial\mu^2}+\beta(g)\frac{\partial}{\partial g}\right)$

Leading Order

The LO DGLAP splitting functions:

$$P_{qq}^{(0)} = C_F \left(\frac{1+x^2}{1-x}\right)_+$$

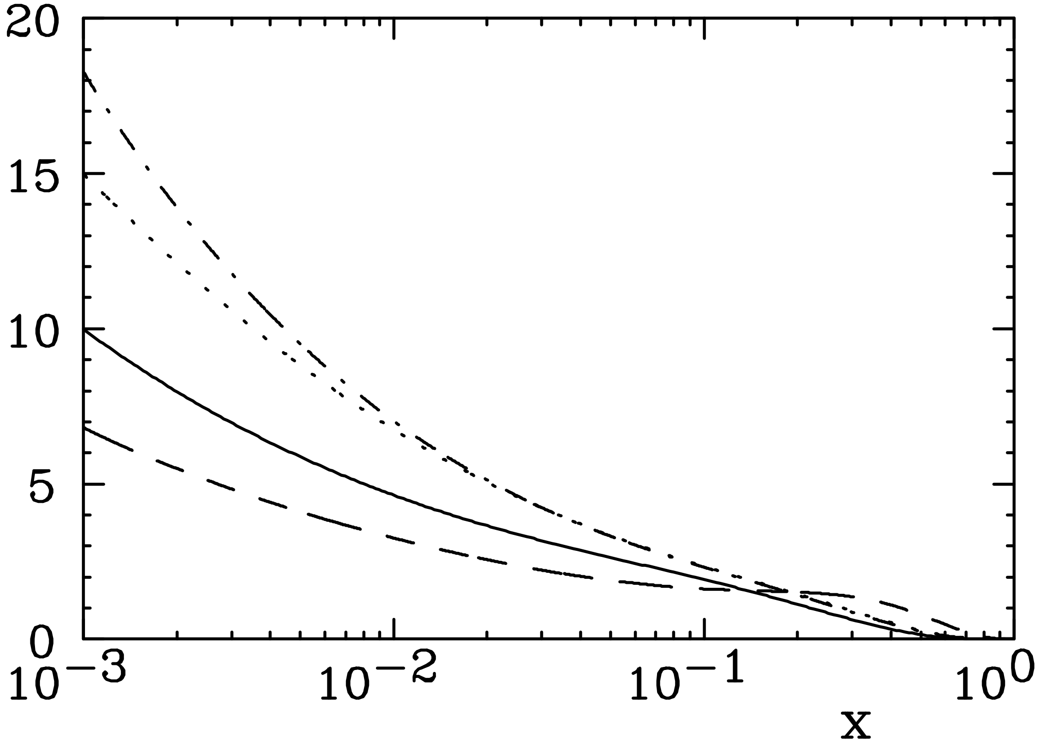
$$\Delta P_{qq}^{(0)} = P_{qq}^{(0)} \quad \text{helicity conservation}$$

$$\Delta_T P_{qq}^{(0)} = C_F \left[\left(\frac{1+x^2}{1-x}\right)_+ - 1 + x\right]$$

$$= P_{qq}^{(0)}(x) - C_F(1-x)$$

N.B. For both $P_{qq}^{(0)}$ and $\Delta P_{qq}^{(0)}$ the first moments vanish (conservation laws and sum rules) ... but not for $\Delta_T P_{qq}^{(0)}$

LO Evolution



Evolution of u-quark helicity and transversity.

Input $\Delta_T u = \Delta u$ at $Q_0^2 = 0.23\,\mathrm{GeV}^2$ (dashed).
The solid (dotted) curve is $\Delta_T u$ (Δu) at $Q^2 = 25\,\mathrm{GeV}^2$.

The dot–dashed curve is evolution of $\Delta_T u$ at $Q^2 = 25\,\mathrm{GeV}^2$ driven by P_{qq}.

Next-to-Leading Order

The situation at NLO is a little more complicated:

Still no quark–gluon mixing (same reasons)

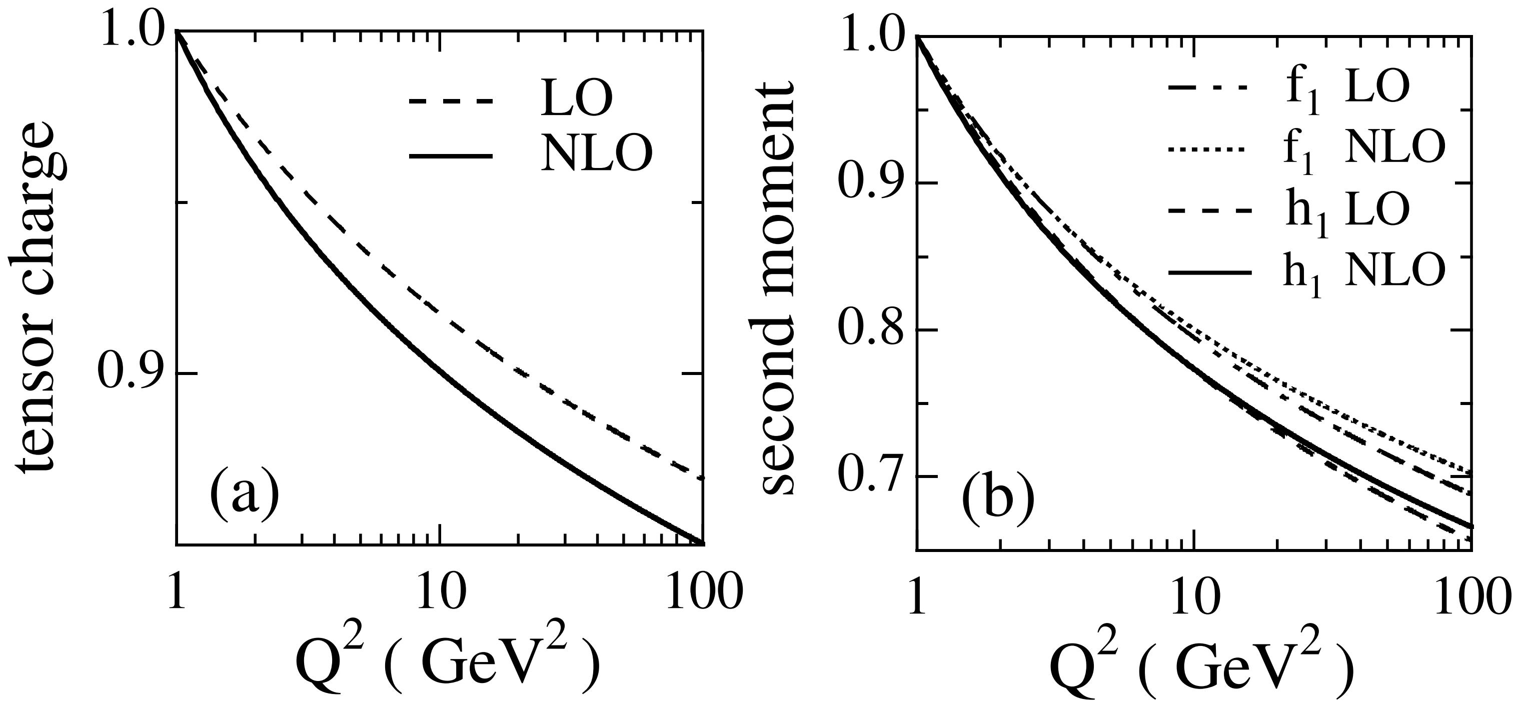
Now have quark–antiquark mixing

And the expressions get much longer . . . and harder to calculate!

The 1-loop coefficient functions are also known for DY

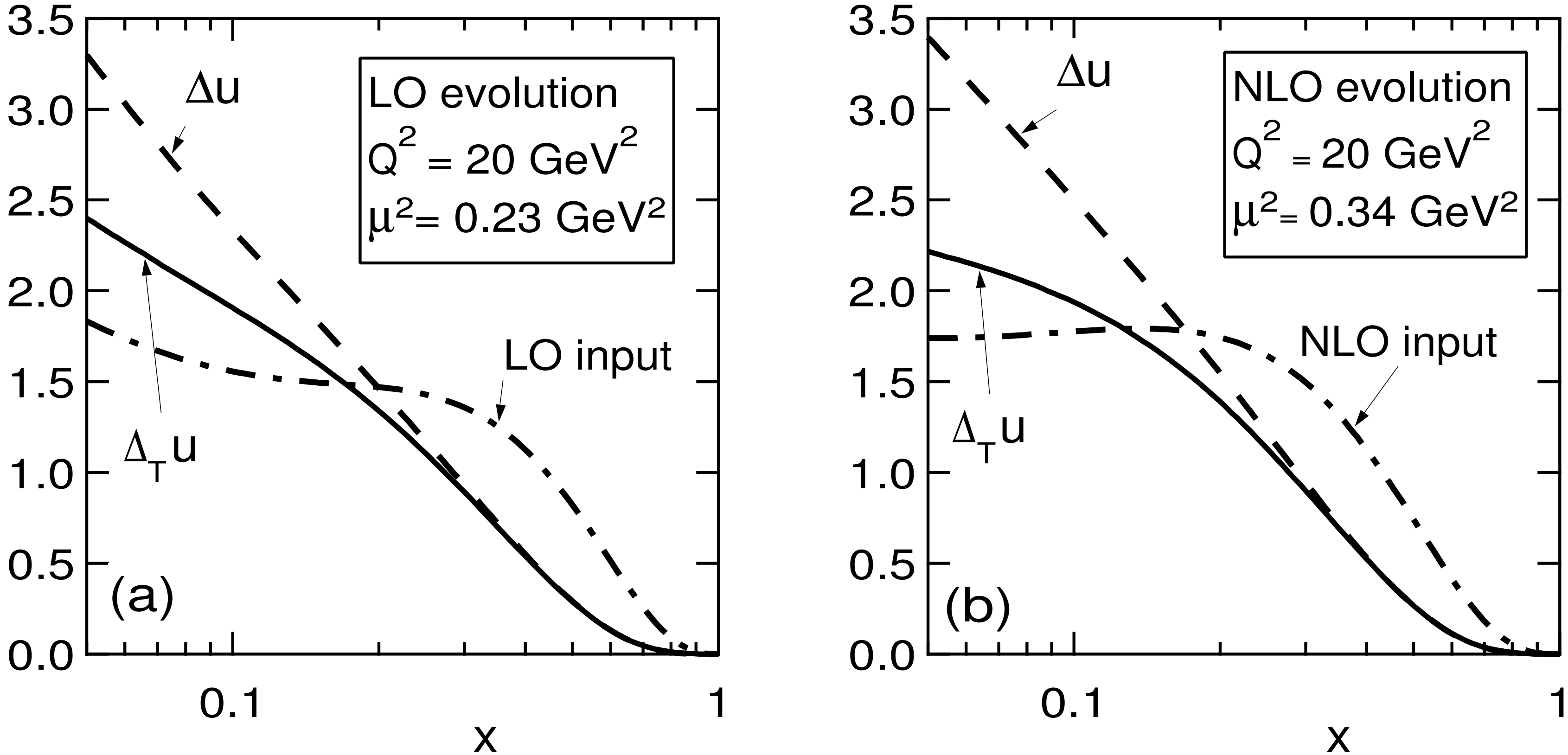
And in different schemes

NLO Evolution



The LO and NLO Q^2-evolution of (a) the tensor charge and (b) the second moments of $h_1(x, Q^2)$ and $f_1(x, Q^2)$ (both normalised at $Q^2 = 1$ GeV^2)

NLO Evolution

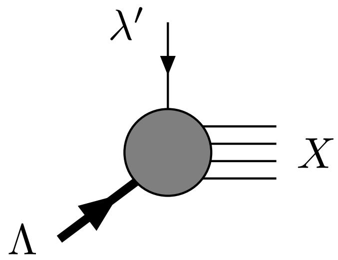


Comparison of the Q^2-evolution of $\Delta_T u(x, Q^2)$ and $\Delta u(x, Q^2)$ at (a) LO and (b) NLO

Soffer Bound

Soffer (1995)

hadron–parton amplitude: $a_{\Lambda,\lambda'} \sim$



$$
\begin{aligned}
f(x) &\propto \mathrm{Im}(\mathcal{A}_{++,++} + \mathcal{A}_{+-,+-}) &&\propto \textstyle\sum_X (a^*_{++}a_{++} + a^*_{+-}a_{+-}) \\
\Delta f(x) &\propto \mathrm{Im}(\mathcal{A}_{++,++} - \mathcal{A}_{+-,+-}) &&\propto \textstyle\sum_X (a^*_{++}a_{++} - a^*_{+-}a_{+-}) \\
\Delta_T f(x) &\propto \mathrm{Im}\,\mathcal{A}_{+-,-+} &&\propto \textstyle\sum_X a^*_{--}a_{++}
\end{aligned}
$$

$$
\sum_X |a_{++} \pm a_{--}|^2 \geq 0 \quad \Rightarrow \quad \sum_X a^*_{++}a_{++} \pm \sum_X a^*_{--}a_{++} \geq 0
$$

leads to

$f_+(x) \geq |\Delta_T f(x)|$ or $\boldsymbol{f(x) + \Delta f(x) \geq 2|\Delta_T f(x)|}$

Evolution of the Soffer Bound

Maintenance of the Soffer bound under QCD evolution has been argued by Bourrely, Leader and Teryaev (1997)
Rewrite the splitting-function "plus" (IR singular) term as

$$P_+(x,t) = P(x,t) - \delta(1-x) \int_0^1 \frac{\mathrm{d}y}{y} \, P(y,t)$$

The DGLAP equations are recast in a Boltzmann form:

$$\frac{\mathrm{d}q(x,t)}{\mathrm{d}t} = \int_x^1 \frac{\mathrm{d}y}{y} \, q(y,t) \, P\left(\frac{x}{y},t\right) - \int_0^x \frac{\mathrm{d}y}{x} \, q(x,t) \, P\left(\frac{y}{x},t\right)$$

The negative term on the RHS is "diagonal" in x and thus cannot change the sign of $q(x,t)$.

Evolution of the Soffer Bound

Now write

$$\frac{\mathrm{d}q_{\pm}(x,t)}{\mathrm{d}t} = P_{+\pm}(x,t) \otimes q_{+}(x,t) + P_{+\mp}(x,t) \otimes q_{-}(x,t)$$

Thus, positivity of the initial distributions, $q_{\pm}(x,t_0) \geq 0$ or $|\Delta q(x,t_0)| \leq q(x,t_0)$, is preserved if both kernels $P_{+\pm}$ are positive, which is true.
The argument can be extended to singlet distributions.
Similarly, consideration of the combinations

$$Q_{\pm}(x) = q_{+}(x) \pm \Delta_T q(x)$$

leads to the evolution safety of the Soffer bound.

A DIS Definition for Transversity

The other twist-2 functions are naturally defined in DIS, where the parton model is usually formulated.

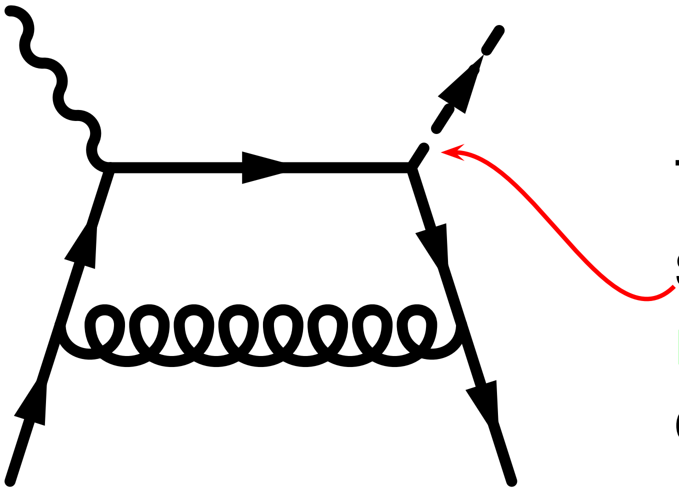
When translated to DY, large K factors appear $\sim \mathrm{O}(\pi\alpha_s)$.

At LEP energies this is a 30% correction, at EMC/SMC energies it is nearly 100%.

Pure DY coefficient functions are known, but are scheme dependent. Moreover, a $\frac{\ln^2 x}{1-x}$ term appears that is not found for spin-averaged or helicity-dependent DY.

Added to the problems arising with the Vector–Scalar current product this suggests an interesting check . . .

DIS Higgs–Photon Interference

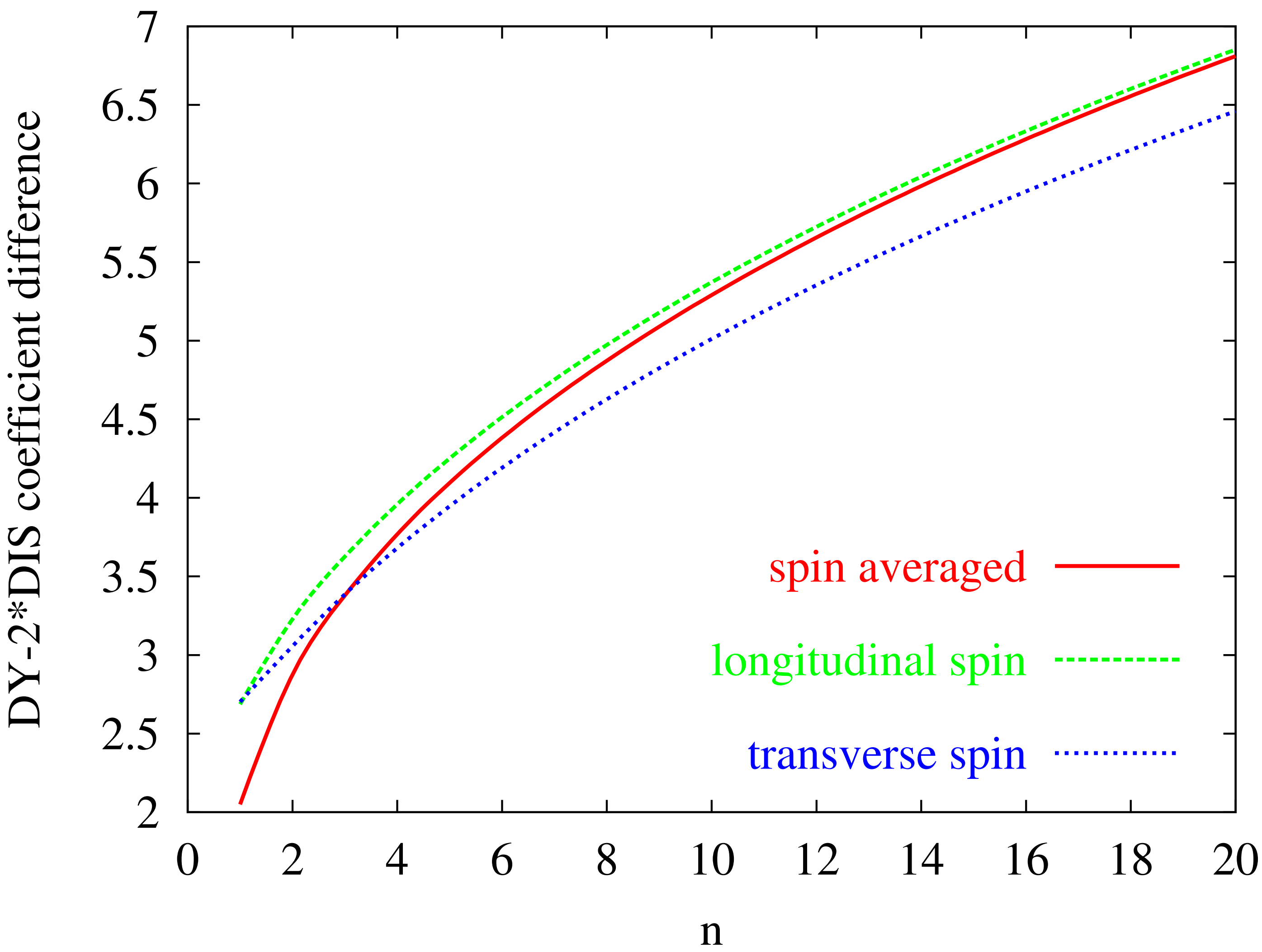


The extra contribution from the scalar vertex is factorised into the running mass (or Higgs coupling constant).

$$C^f_{q,DY} - 2C^f_{q,DIS} = \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\frac{3}{(1-z)_+} + 2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 6 - 4z + \left(\frac{4}{3}\pi^2 + 1\right)\delta(1-z)\right]$$

$$C^h_{q,DY} - 2C^h_{q,DIS} = \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\frac{3z}{(1-z)_+} + 4z\left(\frac{\ln(1-z)}{1-z}\right)_+ - 6z\frac{\ln^2 z}{1-z} + 4(1-z) + \left(\frac{4}{3}\pi^2 - 1\right)\delta(1-z)\right]$$

DIS-DY Coefficient Function



Comments and Conclusions

- Non-singlet & non-mixing behaviour render transversity surprisingly simpler and transparent to study
- At high energies the evolution suppresses $\Delta_T q$ with respect to Δq and q so first measurements are best performed at lower Q^2
- Turning this all on its head, transversity is a wonderful place to study evolution—even the first moment evolves rapidly and there are no uncertainties from gluon densities

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