Sructure Functions

and

Lattice - QCD

- hope : lattice-QCD → complementary 'experiments'

  real experiments $\xrightarrow{\text{test}}$ lattice QCD

  additional information $\xleftarrow{\text{provides}}$ lattice QCD

- How to calculate $\langle x\rangle_u$, $\langle x\rangle_d$, ...
  on the lattice
  Problems and perspectives

- The combination of lattice QCD and χPT

- Testing the instanton phenomenology on the lattice

  → the Bochum - St. Petersburg model

QCD = all you can learn from the QCD generating functional

$$Z[J^a_\mu, \dots] = \frac{1}{N} \int \mathcal{D}[A^a_\mu, \bar{\psi}_i, \psi_i] \cdot e^{i \int d^4x \left\{ \mathcal{L}_{QCD} + J^a_\mu A^{a\mu} + \bar{\eta}_i \psi_i \dots \right\}}$$

quantum field theory

analytic continuation
$t \longleftrightarrow i\tau$

thermodynamics

all static properties of QCD are contained in the partition function (time → temperature)

$$Z = \frac{1}{N} \int \mathcal{D}[A^a_\mu, \bar{\psi}_i, \psi_i]\, e^{-S_{QCD}}$$

$\cdot$ (explicite source/operator terms)

◇ Lattice QCD:

The complete partition function of QCD can be simulated on a lattice up to corrections of higher order in the lattice constant a.

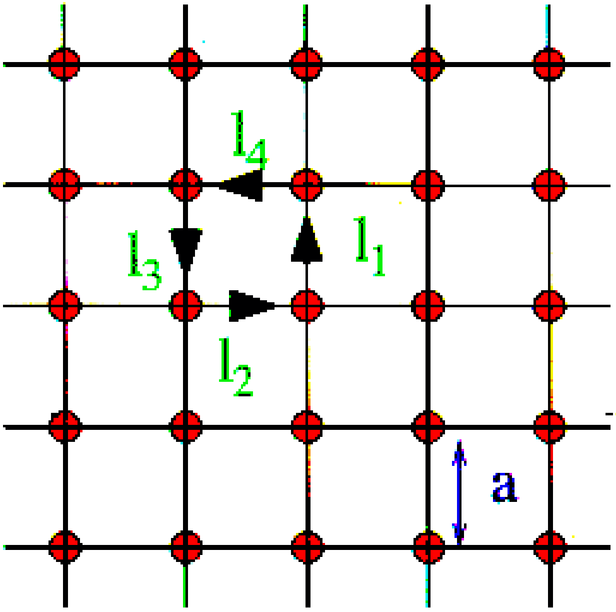


Figure 1: Definition of the links and the lattice spacing

$$
\begin{aligned}
U(l_1) &= \exp\left(-igA^b(l_1)\frac{\lambda^b}{2}a\right) \\
W_\square &= \mathrm{tr}\left\{U(l_1)U(l_2)U(l_3)U(l_4)\right\} \\
\sum_\square \frac{2}{g^2}(3 - W_\square) &= \frac{1}{4}\int d^4x F^a_{\mu\nu}F^{a\mu\nu} \; + \; O(a^5)
\end{aligned}
$$

Source terms:

combinations of quark fields with the appropriate quantum numbers, e.g.

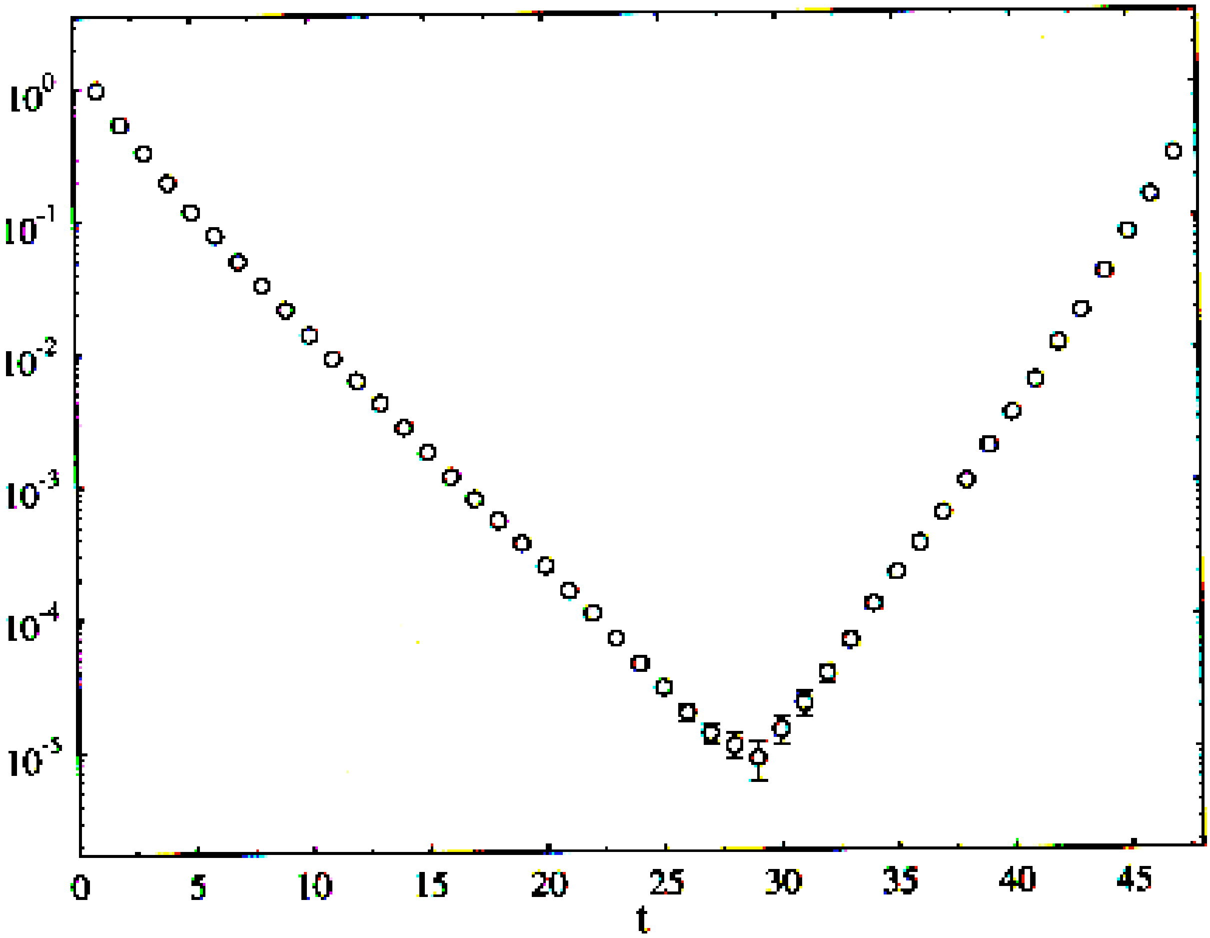
$$B = \epsilon^{ijk} \left[u^i C \gamma_5 d^j \right] u^k$$ plus many tricks, e.g. smearing

operators:

whatever you are interested in, e.g. the operators from OPE related to moments of distribution functions improved

$\frac{1}{2}^+$ $\frac{1}{2}^-$

N N^*



To filter out the information on one specific hadron, i.e. the nucleon, one studies the 2-point function (in Euklidian time)

$$
\begin{aligned}
\langle B(t)B(0)\rangle = & \quad \mathrm{tr}\left\{e^{-H(T-t)}B(t)e^{-Ht}\bar{B}(0)\right\} \\
= & \quad \langle 0|\bar{B}|N\rangle\langle N|B|0\rangle e^{-E_N t} \\
& +\langle 0|\bar{B}|N^*\rangle\langle N^*|B|0\rangle e^{-E_{N^*}t} + \ldots
\end{aligned}
$$

and the 3-point functions

$$
\langle B(t)O(\tau)B(0)\rangle = \langle 0|\bar{B}|N\rangle\langle N|O|N\rangle\langle N|B|0\rangle e^{-E_N t} + \ldots
$$

The exponential slope of the 2-point function gives the proton mass, the ratio of 3-point functions and 2-point function gives the expectation value of the operators O which sre of interest for a given problem (in the limit $\tau, t, T \to \infty$). T is the total size of the lattice in Eukllidian time direction. t and $\tau < t$ are intermediate points. $B(t)$ is a source respectively sink term, i.e. a combination of quark-fields with the correct quantum numbers.

◇ Due to the specific symmtries realized on the lattice the two 2-point function for a baryon is not symmetric for times between t and $T-t$ one observes the parity partner of the nucleon, i.e. the $N^*(1535)$ respectively $N^*(1650)$. So far we always disregarded it.

The usual OPE:

$$\int_0^1 g_1(x,Q^2)\,dx = \frac{1}{2}a_0 + \frac{m^2}{9Q^2}\left(a_2 + 4d_2 + 4f_2\right) + \dots$$

$$\int_0^1 g_2(x,Q^2)\,dx = 0 + O(1/Q^4)$$

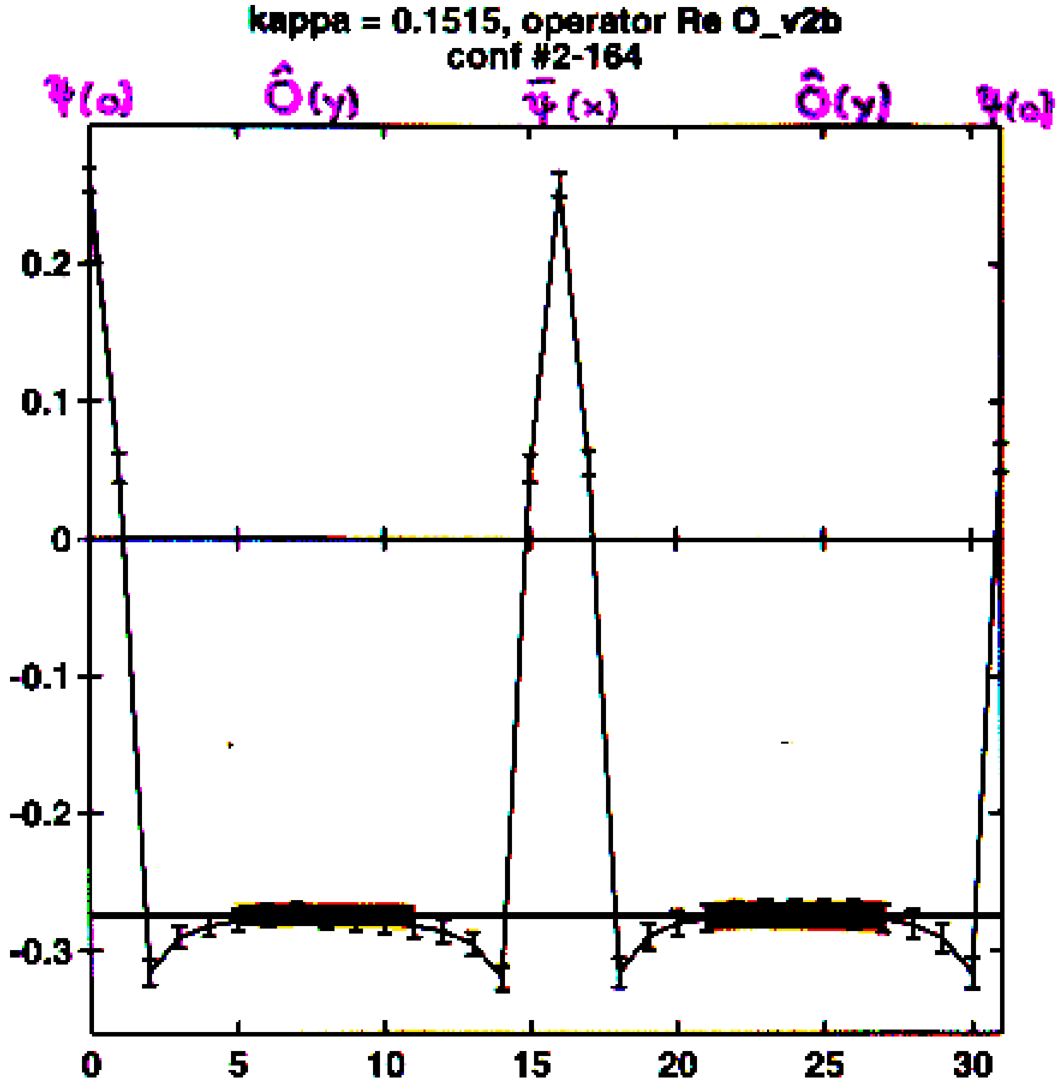
$$\int_0^1 g_1(x,Q^2)\,x^2dx = \frac{1}{2}a_2 + O(1/Q^2)$$

$$\int_0^1 g_2(x,Q^2)\,x^2dx = -\frac{1}{3}a_2 + \frac{1}{3}d_2 + O(1/Q^2)$$

$$\frac{1}{6}\langle PS|\,\bar{q}_f(0)\left(\gamma^\alpha g\tilde{G}^{\beta\sigma} + \gamma^\beta g\tilde{G}^{\alpha\sigma}\right)q_f(0)\,|PS\rangle - \text{traces}$$

$$= 2d_{2f}\Big[\frac{1}{6}\big(2p^\alpha p^\beta S^\sigma + 2p^\beta p^\alpha S^\sigma - p^\beta p^\sigma S^\alpha - p^\alpha p^\sigma S^\beta - p^\sigma p^\alpha S^\beta - p^\sigma p^\beta S^\alpha\big) - \text{traces}\Big]$$

$$\langle PS|\,\bar{q}_f(0)\,g\tilde{G}_{\alpha\beta}\gamma^\beta q_f(0)\,|PS\rangle = 2M^2 f_{2f}S_\alpha$$



$\sim \int x\, u_v(x)\, dx$ π

Ch. Best
S. Schramm

Example: The spin structure of the $\Lambda^{\uparrow}$

SU(6) quark model:

$$\Lambda^{\uparrow} \sim s^{\uparrow} \qquad \Delta s_{\Lambda} = 1 \quad \Delta d_{\Lambda} = \Delta u_{\Lambda} = 0$$

all the spin should be carried by the s-quark

exp. data on $p^{\uparrow}, n^{\uparrow}$ + $SU(3)_F$ rotation

$$\Delta s_{\Lambda} = 0.63(3) \qquad \Delta u_{\Lambda} = \Delta d_{\Lambda} = -0.17(3)$$

Ashery + Lipkin

Lattice : QCDSF Göckeler et al. Lattice 2002

$$\Delta u_{\Lambda} = \Delta d_{\Lambda} = -0.01(4) \qquad \Delta s_{\Lambda} = 0.67(3)$$

lattice nucleon + $SU(3)_F$: $\Delta u_{\Lambda} = \Delta d_{\Lambda} = -0.016(9)$; $\Delta s_{\Lambda} = 0.65(2)$

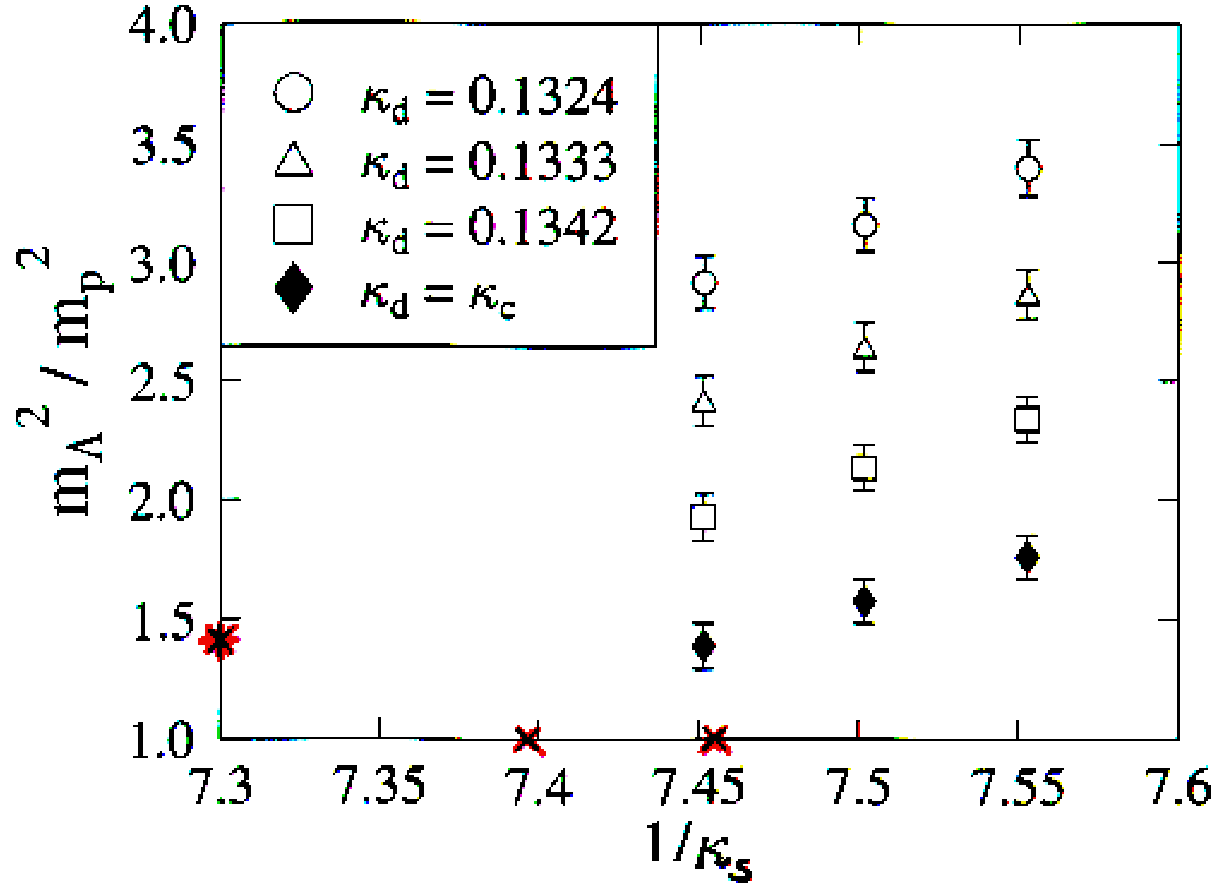
The effect of m_s on matrix elements seems to be weaker than on the Λ mass.

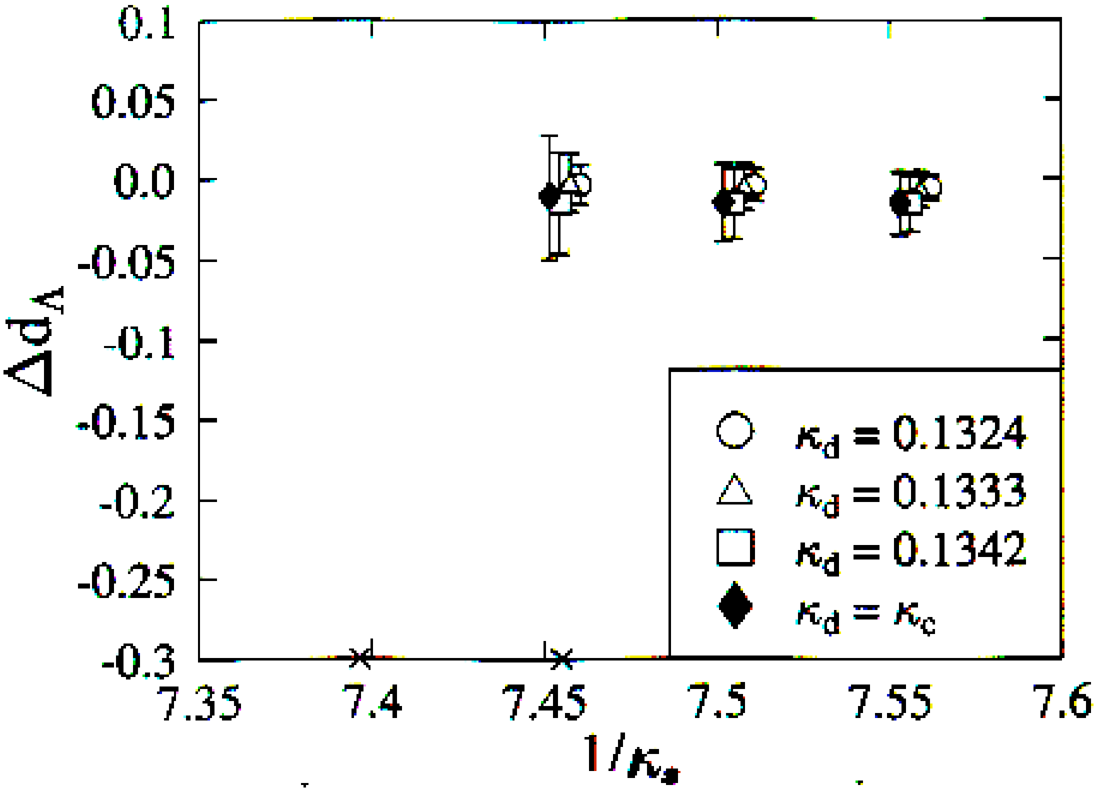
Wilson Fermions : $\kappa \longleftrightarrow m_q$

$\kappa \to \kappa_{cr}$: $m_q \to 0$

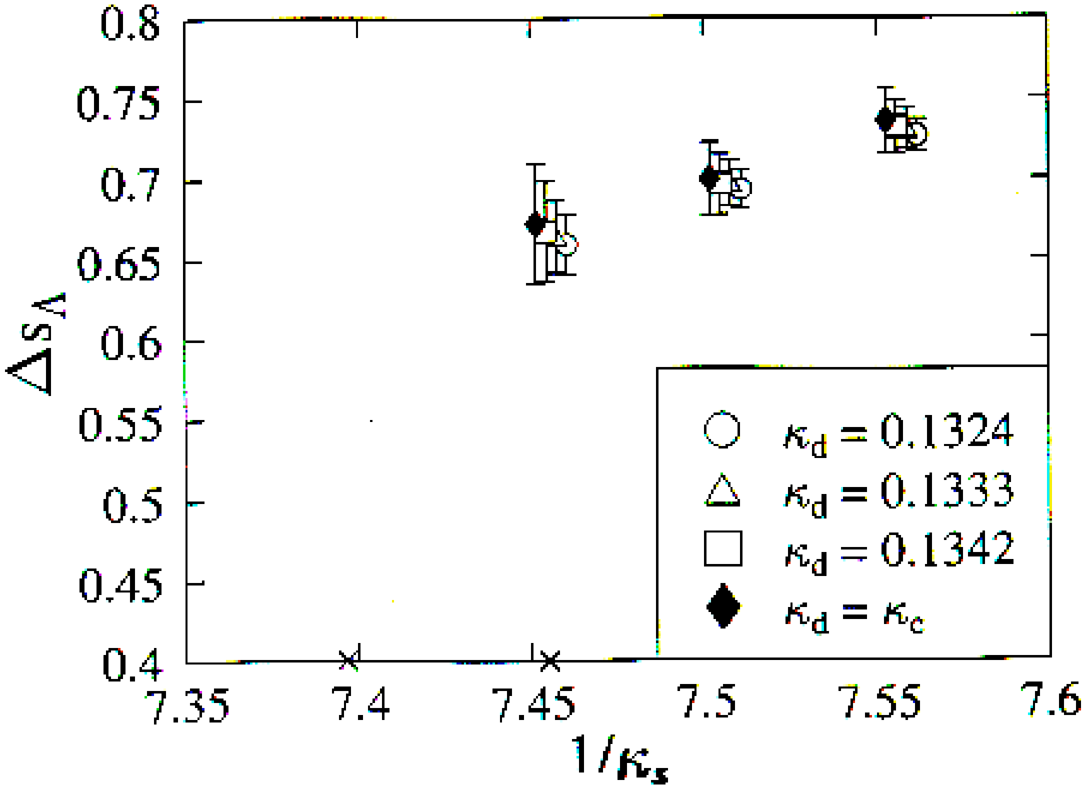
calculations become more and more expensive

$1/\kappa_s = 7.45$ ~ physical masses





0.1
0.05
0.0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
7.35
7.4
7.45
7.5
7.55
7.6
Δd_Λ
$1/\kappa_s$
$\kappa_d = 0.1324$
$\kappa_d = 0.1333$
$\kappa_d = 0.1342$
$\kappa_d = \kappa_c$



0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
7.35
7.4
7.45
7.5
7.55
7.6
Δs_Λ
$1/\kappa_s$
$\kappa_d = 0.1324$
$\kappa_d = 0.1333$
$\kappa_d = 0.1342$
$\kappa_d = \kappa_c$

LHPC + SESAM

MIT, Wuppertal, ... hep-lat/0201021

Connected M. E.	QCDSF	QCDSF ($a=0$)	Wuppertal	Quenched	Full QCD (3 pts)	Phenomenology ($q \pm \bar{q}$)
$\langle x \rangle_u$	0.452(26)			0.454(29)	0.459(29)	
$\langle x \rangle_d$	0.189(12)			0.203(14)	0.190(17)	
$\langle x \rangle_{u-d}$	0.263(17)			0.251(18)	0.269(23)	0.154(3) ?
$\langle x^2 \rangle_u$	0.104(20)			0.119(61)	0.176(63)	
$\langle x^2 \rangle_d$	0.037(10)			0.029(32)	0.031(30)	
$\langle x^2 \rangle_{u-d}$	0.067(22)			0.090(68)	0.145(69)	0.055(1)
$\langle x^3 \rangle_u$	0.022(11)			0.037(36)	0.069(39)	
$\langle x^3 \rangle_d$	$-0.001(7)$			0.009(18)	$-0.010(15)$	
$\langle x^3 \rangle_{u-d}$	0.023(13)			0.028(49)	0.078(41)	0.023(1)
$\langle 1 \rangle_{\Delta u}$	0.830(70)	0.889(29)	0.816(20)	0.888(80)	0.860(69)	
$\langle 1 \rangle_{\Delta d}$	$-0.244(22)$	$-0.236(27)$	$-0.237(9)$	$-0.241(58)$	$-0.171(43)$	
$\langle 1 \rangle_{\Delta u - \Delta d}$	1.074(90)	1.14(3)	1.053(27)	1.129(98)	1.031(81)	1.248(2) ?
$\langle x \rangle_{\Delta u}$	0.198(8)			0.215(25)	0.242(22)	
$\langle x \rangle_{\Delta d}$	$-0.048(3)$			$-0.054(16)$	$-0.029(13)$	
$\langle x \rangle_{\Delta u - \Delta d}$	0.246(9)			0.269(29)	0.271(25)	0.196(9)
$\langle x^2 \rangle_{\Delta u}$	0.087(14)			0.027(60)	0.116(42)	
$\langle x^2 \rangle_{\Delta d}$	$-0.025(6)$			$-0.003(25)$	0.001(25)	
$\langle x^2 \rangle_{\Delta u - \Delta d}$	0.112(15)			0.030(65)	0.115(49)	0.061(6)
δu_c	0.93(3)	0.980(30)		1.01(8)	0.963(59)	
δd_c	$-0.20(2)$	$-0.234(17)$		$-0.20(5)$	$-0.202(36)$	
d_2^u	$-0.206(18)$			$-0.233(86)$	$-0.228(81)$	
d_2^d	$-0.035(6)$			0.040(31)	0.077(31)	

TABLE IX. Comparison of linear extrapolations of full QCD and quenched results with other lattice calculations and phenomenology at 4 GeV in the $\overline{MS}$ scheme. The first column shows quenched results by the QCDSF collaboration at $\beta = 6.0$ [10,50–52] and the second column shows extrapolation of several moments to the continuum limit [15]. The third column shows full QCD results calculated using a different method with the same SESAM configurations we have used [14]. The quenched and full QCD results calculated in this work are shown in the fourth and fifth columns. Flavor non-singlet moments $\langle x^n \rangle$ of $q(x) + (-1)^{n+1}\bar{q}(x)$, $\Delta q(x) + (-1)^n \Delta\bar{q}(x)$, and $\delta q(x) + (-1)^{n+1}\delta\bar{q}(x)$ are tabulated in the final column. Phenomenological unpolarized distributions are calculated from refs. [1–3] and polarized distributions are calculated from refs. [4,5] with error estimates as described in the text.

For the nucleon structure functions

phenomenological success

was limited so far !

- extrapolation to the chiral limit
  + chiral perturbation theory

- finite size artefacts

- chirally improved fermions

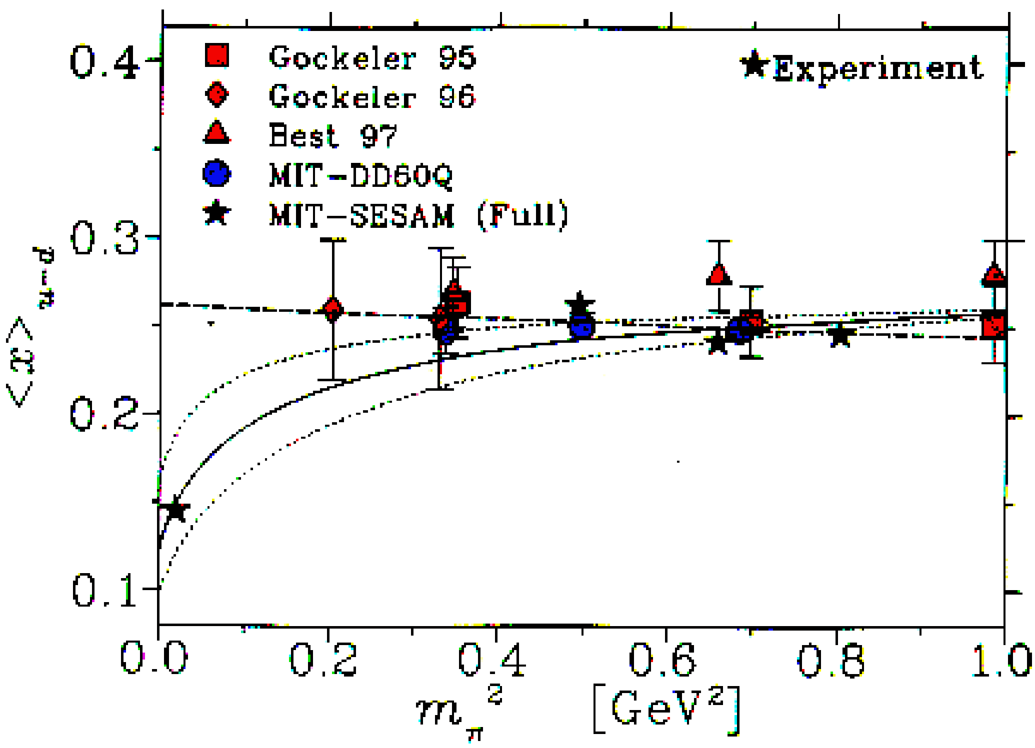
  ...

But:

You can extrapolate to a world with heavier up / down quarks and pions with χPT

→ The linear extrapolation normally used is not reliable

A. Thomas, J. Negele
W. Detmold, W. Melnitchouk ...



Gockeler 95
Gockeler 96
Best 97
MIT-DD60Q
MIT-SESAM (Full)
Experiment
0.4
0.3
0.2
0.1
$\langle x \rangle_{u-d}$
0.0
0.2
0.4
0.6
0.8
1.0
m_π^2 [GeV^2]

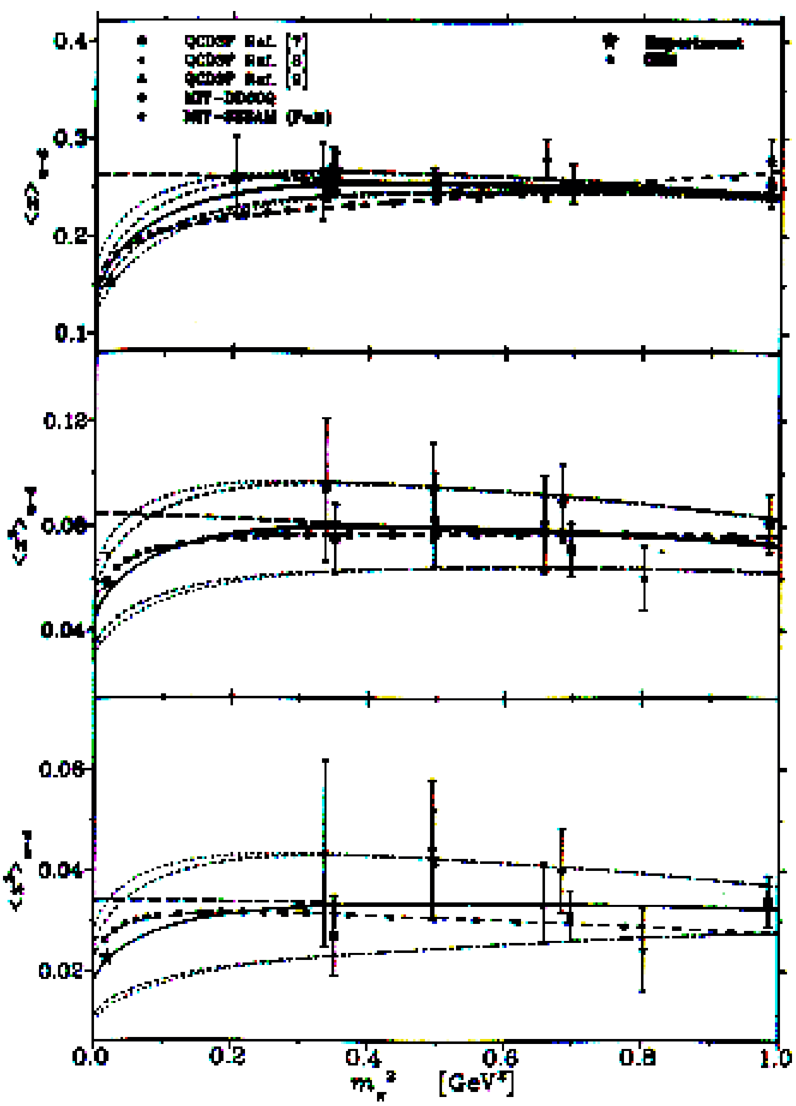
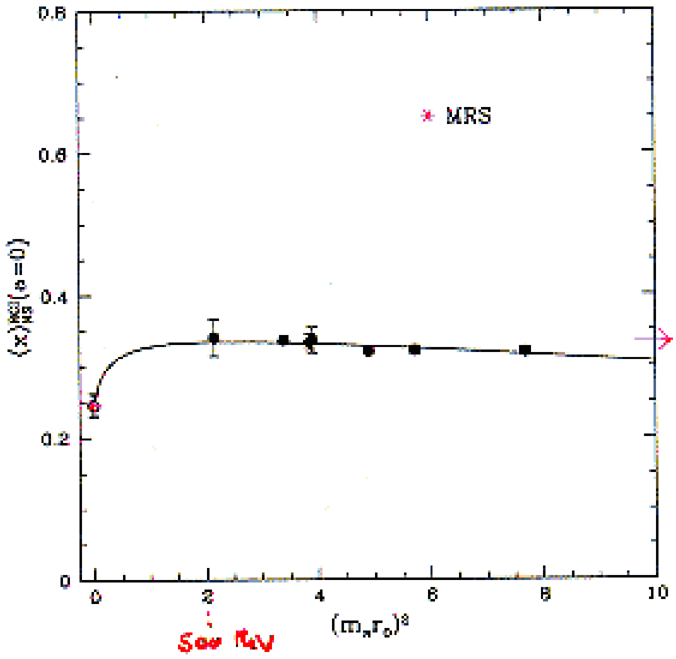
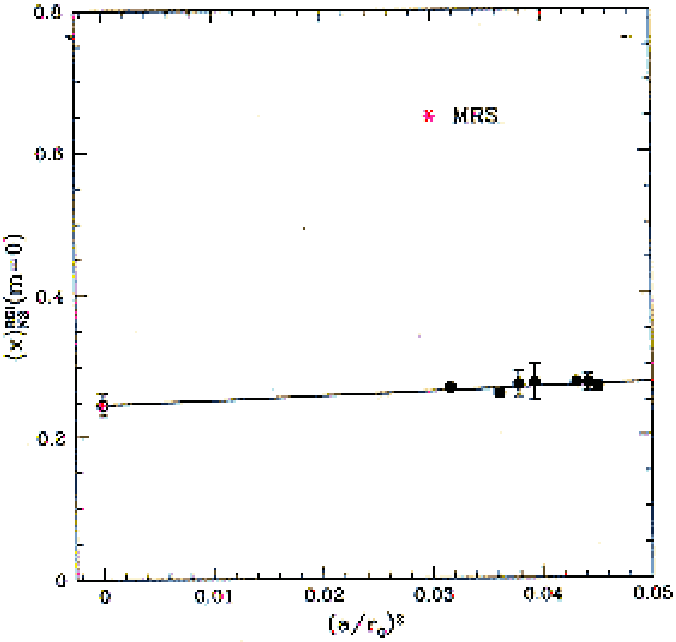


FIG. 1. Moments of the $u-d$ quark distribution. The straight (long-dashed) lines are linear fits to the data, while the curves have the correct LNA behavior in the chiral limit. For each moment, the best fit to the lattice data using Eq.(3) is shown by the solid curve (with $\mu = 550$ MeV), while the inner envelope about this represents the statistical errors in the data. The best fit parameters are: $a_1 = 0.1364$, $b_1 = -0.0648\,\mathrm{GeV}^{-2}$, $a_2 = 0.0438$, $b_2 = -0.0252\,\mathrm{GeV}^{-2}$, $a_3 = 0.0176$, $b_3 = -0.00693\,\mathrm{GeV}^{-2}$, which give a χ^2 per degree of freedom of 0.9, 0.5 and 0.5 for $n = 1$, 2 and 3, respectively. The effect of the uncertainty in the parameter μ is illustrated by the outer lower (upper) short-dashed curves, which correspond to $\mu = 450$ (650) MeV. The small squares are the CBM results, and the dashed curve through them best fits using Eq.(3). The star represents the phenomenological values taken from NLO fits [14] in the $\overline{\mathrm{MS}}$ scheme.

$\langle x \rangle$ Full QCD

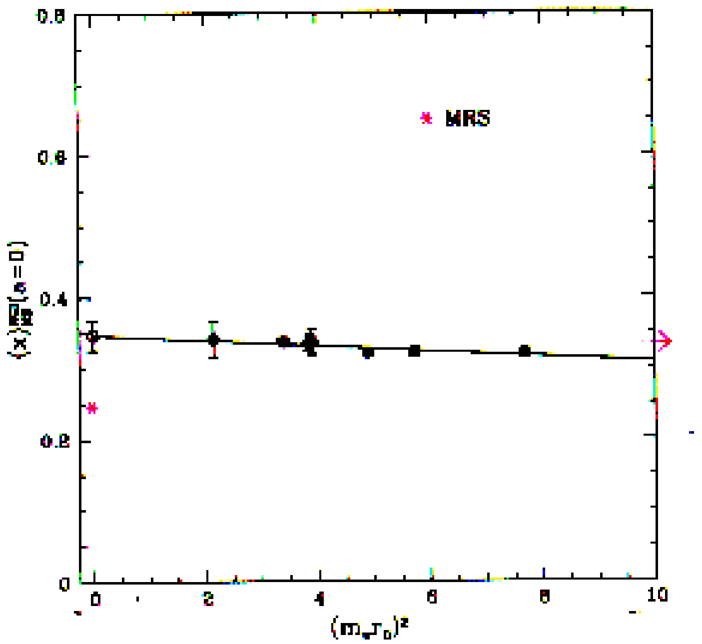


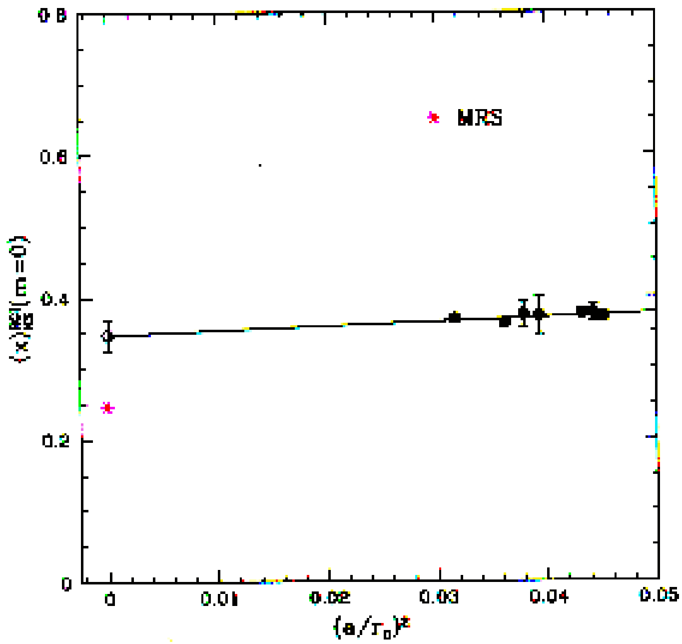


$$\langle x \rangle = A\left(1 - \frac{3g_A^2+1}{(4\pi f_\pi)^2} m_\pi^2 \ln\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)\right) + B(m_\pi r_0)^2 + C(a/r_0)^2$$

$\Lambda \approx 350$ MeV Thomas et al., Arndt & Savage, Chen & Ji

$\langle x \rangle$ Full QCD   NS, RGI

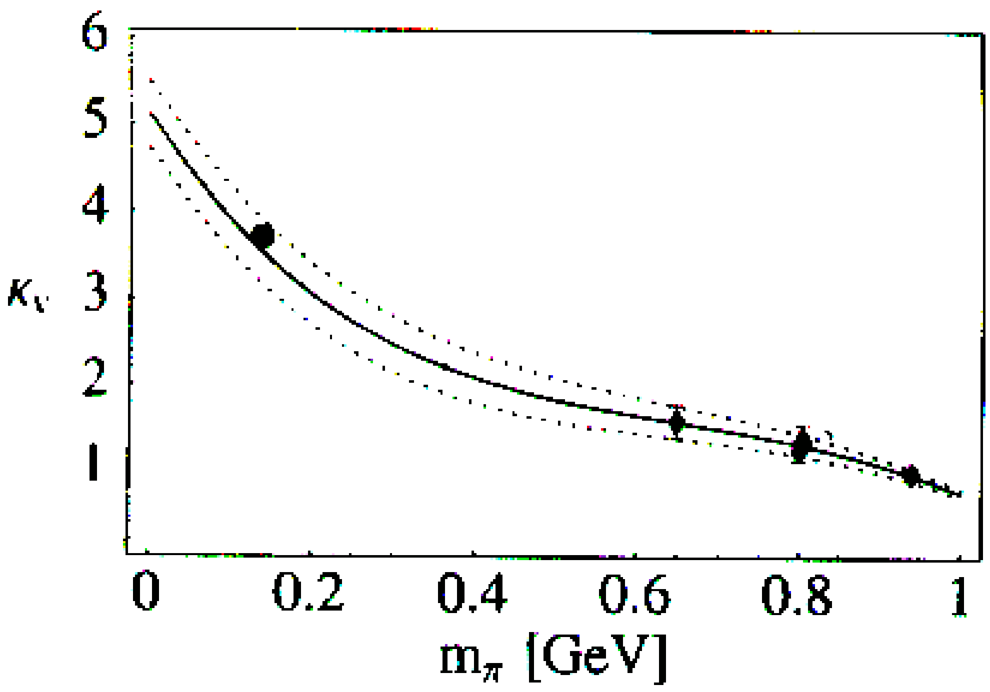


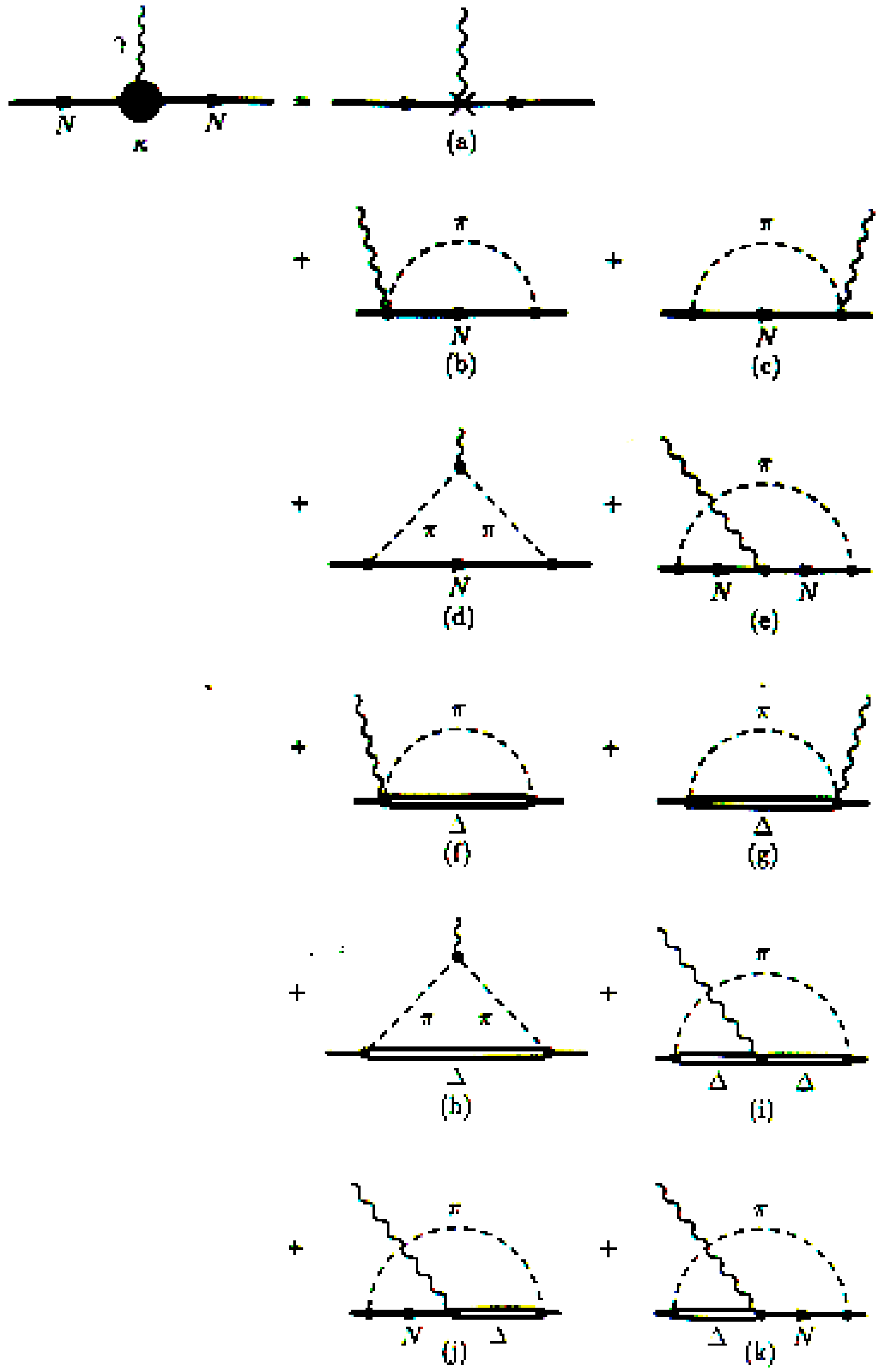


Fit ansatz: $\langle x \rangle = A + B(m_\pi r_0)^2 + C(a/r_0)^2$, $r_0 = 0.5$ fm anomalous magnetic moments

$\kappa_p - \kappa_n$

Hemmert + Weise + ...

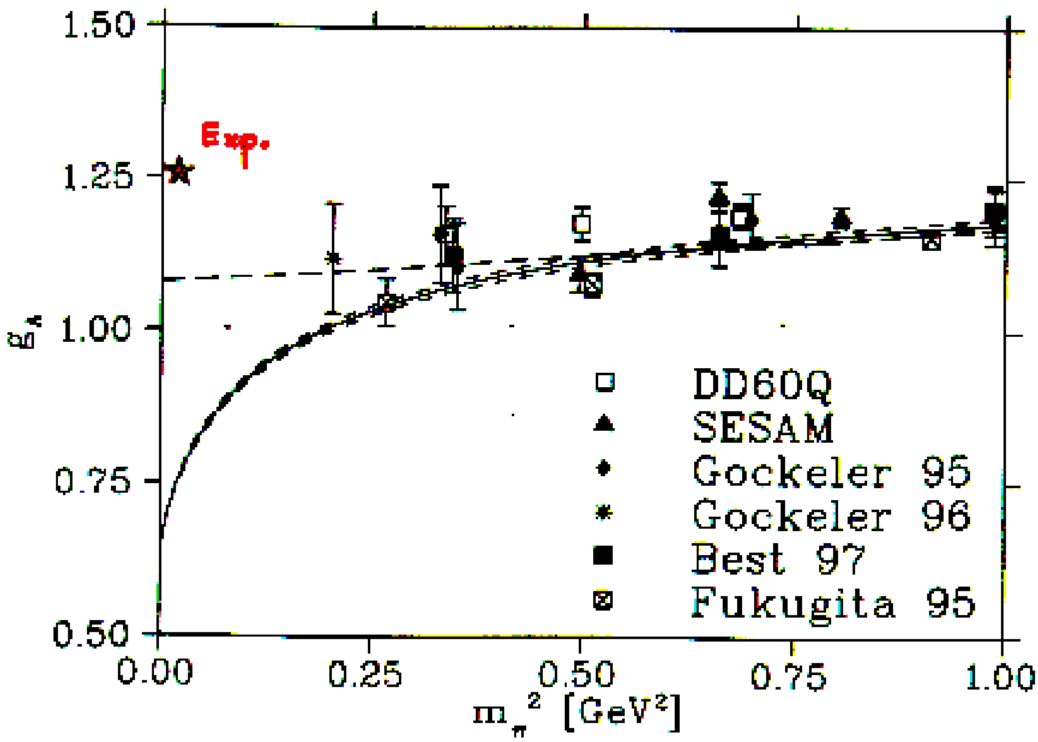



γ
N
κ
N
(a)
π
N
(b)
π
N
(c)
π
π
N
(d)
π
N
N
(e)
π
Δ
(f)
π
Δ
(g)
π
π
Δ
(h)
π
Δ
Δ
(i)
π
N
Δ
(j)
π
Δ
N
(k)

But :

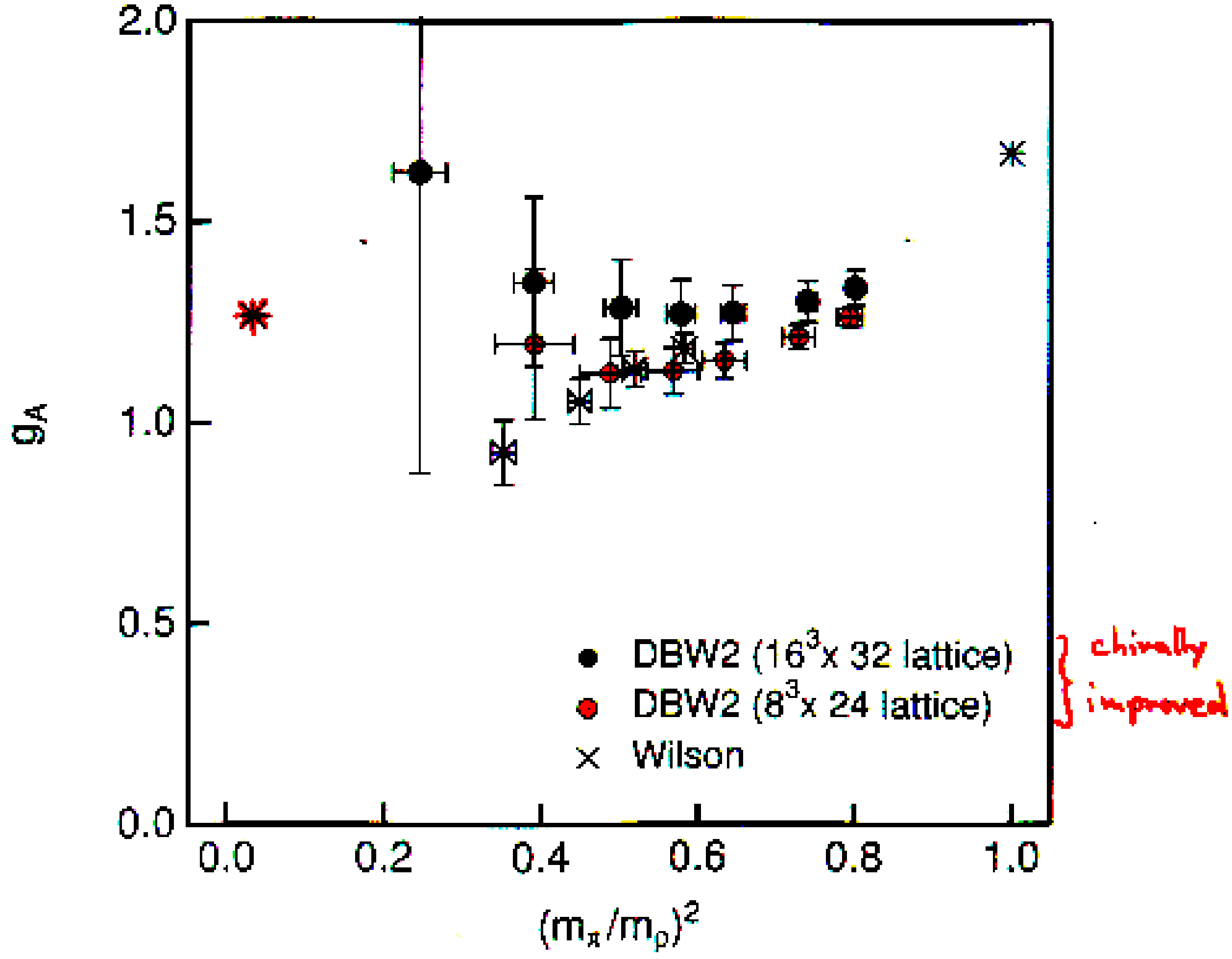
$$g_A = \Delta u_p - \Delta d_p$$

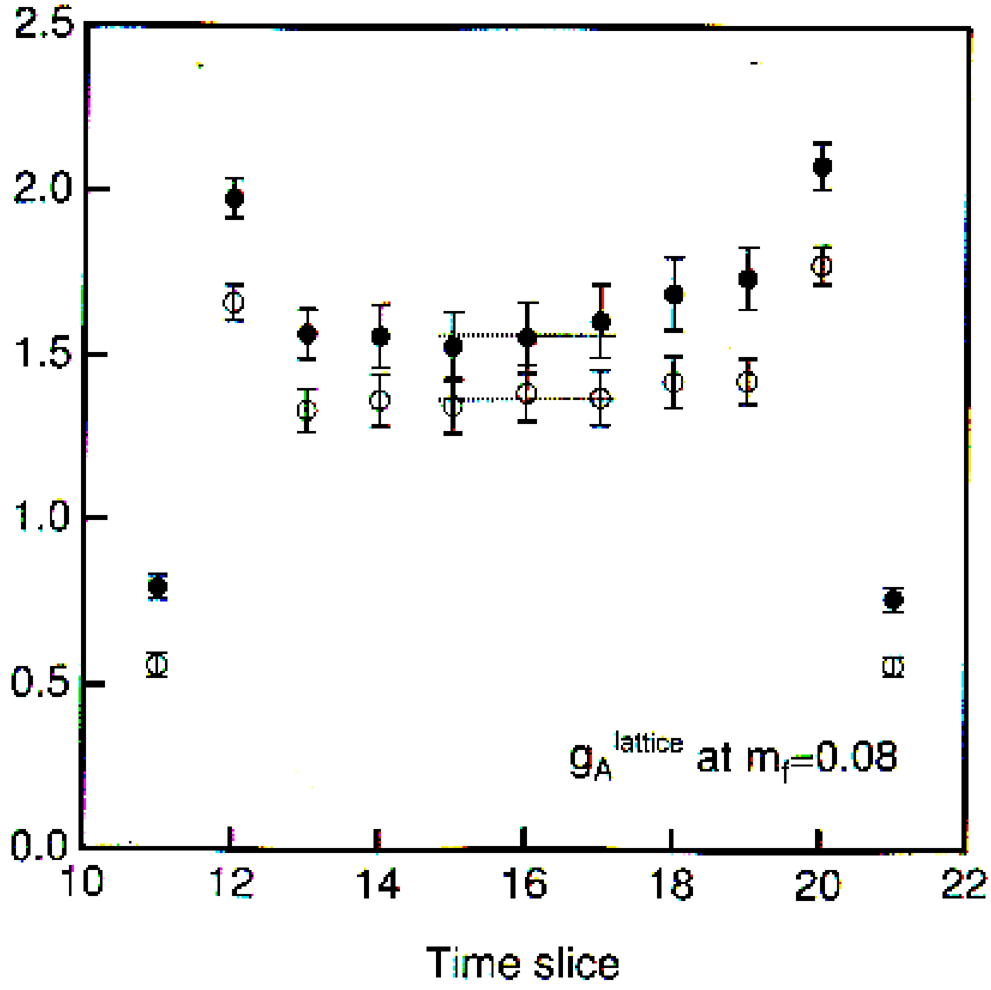
$g_A = \Delta u - \Delta d$



Sasaki et al. hep-lat/0110053 Lattice 2002

You need chirally improved fermions !





2.5
2.0
1.5
1.0
0.5
0.0
10
12
14
16
18
20
22
Time slice
$g_A^{lattice}$ at m_f=0.08

Lattice calculations as

Test for models !

e.g.

Diakonov, Petrov, Wakamatsu, Polyakov, Weiss, Göke, ... :

Instanton based phenomenology

→ predictions for structure functions, GPDs etc.

JLab, Kentucky, Adelaide ... :

"Lattice QCD does not support the instanton liquid model"

Ch. Gattringer et al. (Regensburg) : It does !

The new state of the art :

chiral fermions

Nielsen - Ninomiya theorem:

It is not possible to have simultaneously

- the correct continuum limit
- good chiral properties
- a local Dirac operator $\Longleftrightarrow$ **non-local operators**

on the lattice

$\gamma_5 \not{D} + \not{D} \gamma_5 = 0$ is not possible

next neighbor

$\gamma_5 \not{D} + \not{D} \gamma_5 = a \not{D} \gamma_5 \not{D}$ is possible

$\Rightarrow$ $\not{D}$ is not next neighbor

$\Rightarrow$ needs extremely much CPU time

Nielsen-Ninomiya theorem:

It is not possible to have simultaneously

- the correct continuum limit
- good chiral properties
- a local Dirac operator $\Longleftrightarrow$ **non-local operators**

on the lattice

$\gamma_5 D + D \gamma_5 = 0$ is not possible

next neighbor

$\gamma_5 D + D \gamma_5 = a D \gamma_5 D$ is possible

$\Rightarrow$ D is not next neighbor

$\Rightarrow$ needs extremely much CPU time

◇ Chirally Improved Fermions

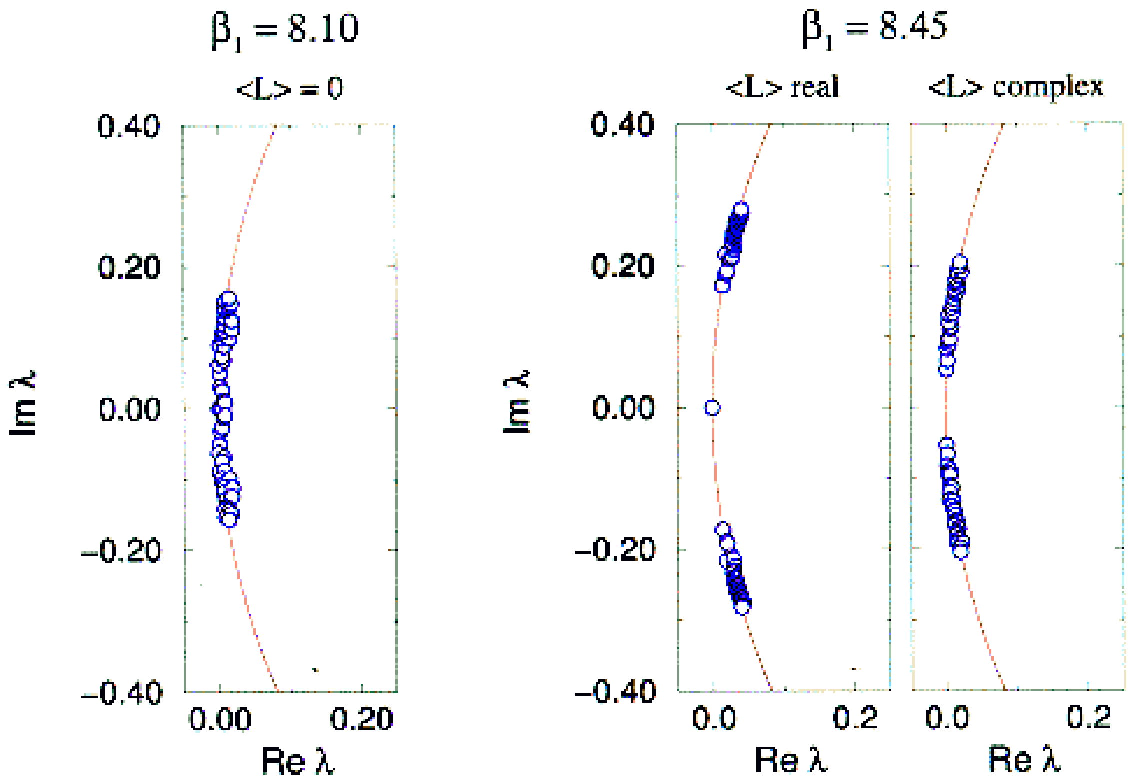
The Ginsparg-Wilson equation for the lattice Dirac operator reads:

$$\gamma_5 D \; + \; D \gamma_5 \; = \; D \gamma_5 D$$

It can be solved approximately making use of the expansion (Ch. Gattringer)

$$\begin{aligned}
D \; & \equiv \mathbb{1}\Big[s_1 <> \; + s_2 \sum_{l_1} < l_1 > \; + s_3 \sum_{l_2 \neq l_1} < l_1, l_2 > \; + s_4 \sum_{l_1} < l_1, l_1 > \; \ldots \Big] \\
& + \; \sum_{\mu} \gamma_\mu \sum_{l_1 = \pm\mu} s(l_1) \Big[v_1 < l_1 > \; + v_2 \sum_{l_2 \neq \pm\mu} [< l_1, l_2 > + < l_2, l_1 >] \\
& \qquad\qquad\qquad\qquad\qquad\qquad + v_3 < l_1, l_1 > \; \ldots \Big] \\
& + \; \sum_{\mu<\nu} \gamma_\mu \gamma_\nu \sum_{\substack{l_1 = \pm\mu \\ l_2 = \pm\nu}} s(l_1)\, s(l_2) \sum_{i,j=1}^{2} \epsilon_{ij} \Big[t_1 < l_i, l_j > \; \ldots \Big] \\
& + \; \sum_{\mu<\nu<\rho} \gamma_\mu \gamma_\nu \gamma_\rho \sum_{\substack{l_1 = \pm\mu, l_2 = \pm\nu \\ l_3 = \pm\rho}} s(l_1)\, s(l_2)\, s(l_3) \sum_{i,j,k=1}^{3} \epsilon_{ijk} \Big[a_1 < l_i, l_j, l_k > \; \ldots \Big] \\
& + \; \gamma_5 \sum_{\substack{l_1 = \pm 1, l_2 = \pm 2 \\ l_3 = \pm 3, l_4 = \pm 4}} s(l_1)\, s(l_2)\, s(l_3)\, s(l_4) \sum_{i,j,k,n=1}^{4} \epsilon_{ijkn} \Big[p_1 < l_i, l_j, l_k, l_n > \; \ldots \Big]
\end{aligned}$$

leading to results of quite satisfactory quality:



The 50 eigenvalues closest to the origin. All spectra are for 6×20^3 lattices.

note: $\langle \bar{\psi}\psi \rangle = -\frac{\pi}{V}\rho(0)$ chiral condensate

there is an exact zero mode for each topological charge

$\langle L \rangle$ = Polyakov loop ⇒ confinement - deconfinement

above T_{cr} it has three possible phases

$e^{i\varphi}$, $\varphi = 0, \pm\frac{2\pi}{3}$ for quenched calculations note $\quad D\gamma_5 = \gamma_5 D^\dagger$

$$\gamma_5 D + D\gamma_5 - D\gamma_5 D = 0$$

$$\Rightarrow \quad D + D^\dagger - D^\dagger D = 0$$

$$2\,\mathrm{Re}\,\lambda - (\mathrm{Im}\,\lambda)^2 - (\mathrm{Re}\,\lambda)^2 = 0$$

$$(\mathrm{Im}\,\lambda)^2 + (\mathrm{Re}\,\lambda - 1)^2 = 1$$

Circle of radius 1 around $\mathrm{Re}\,\lambda = 1$, $\mathrm{Im}\,\lambda = 0$

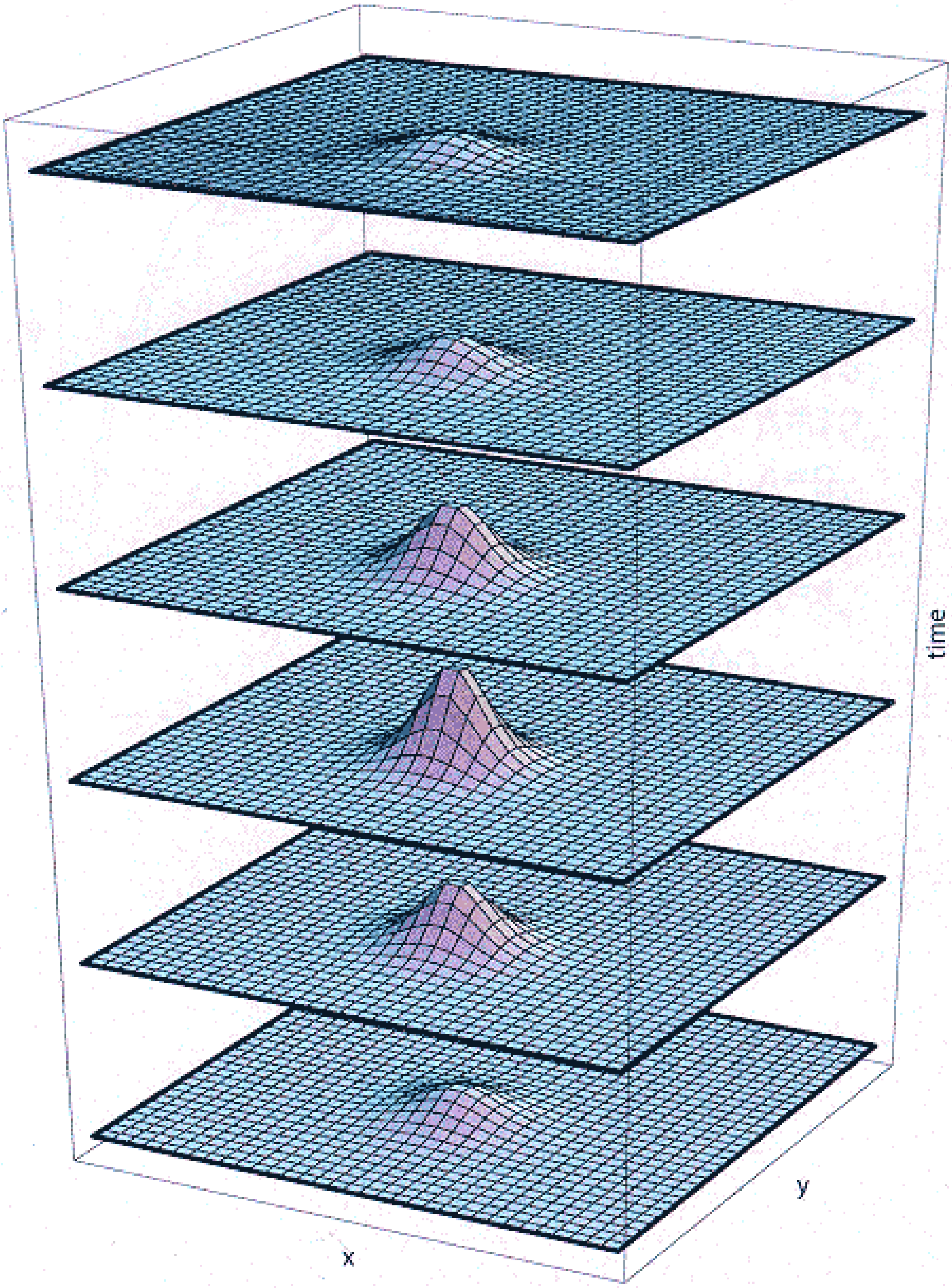
We studied the Dirac-eigenvectors for small eigenvalues.

- For each topological mode there is 1 Dirac eigenstate with eigenvalue zero

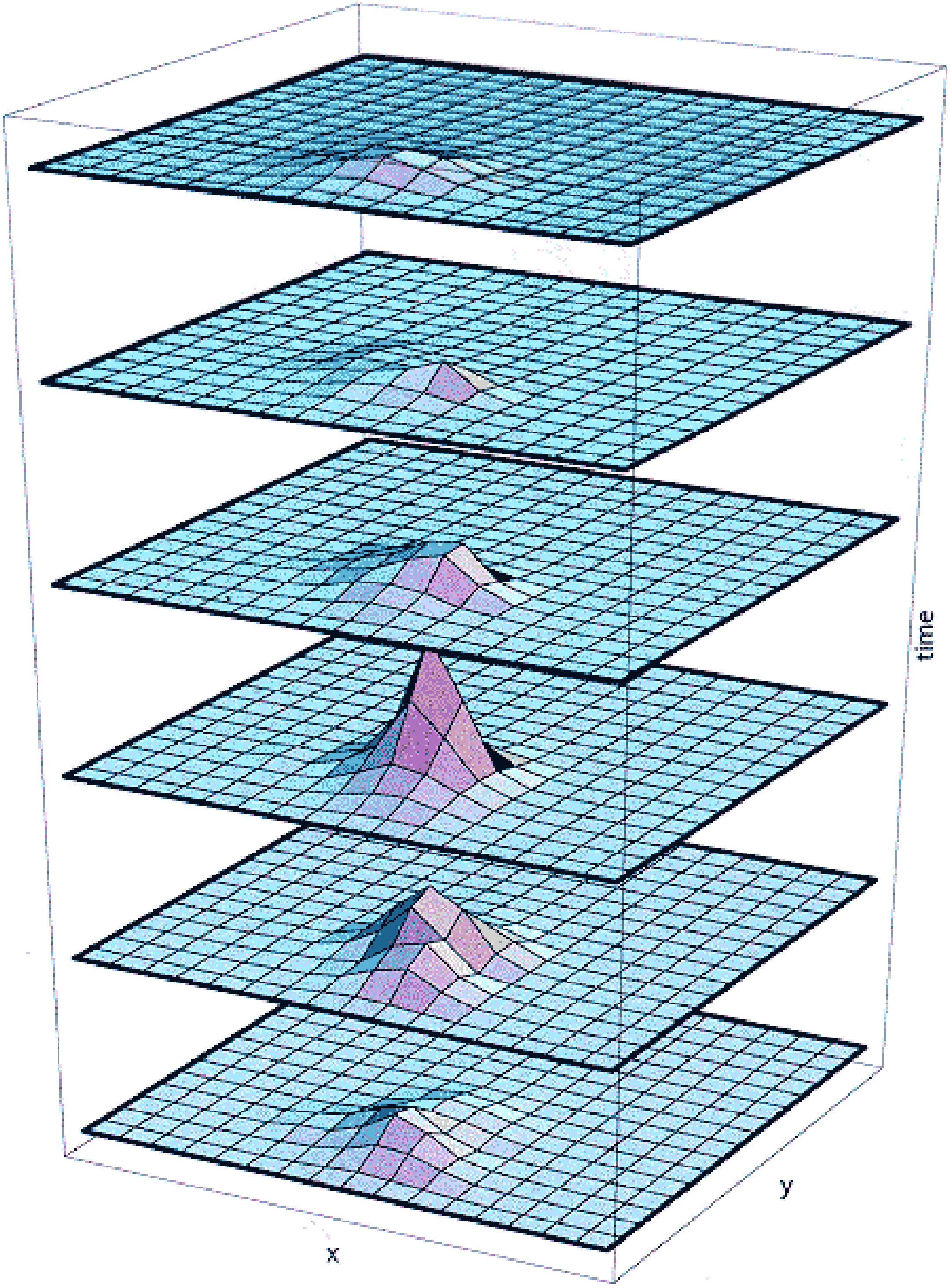
<u>Note:</u>

Chiral condensate $\Longleftrightarrow$ hadron masses

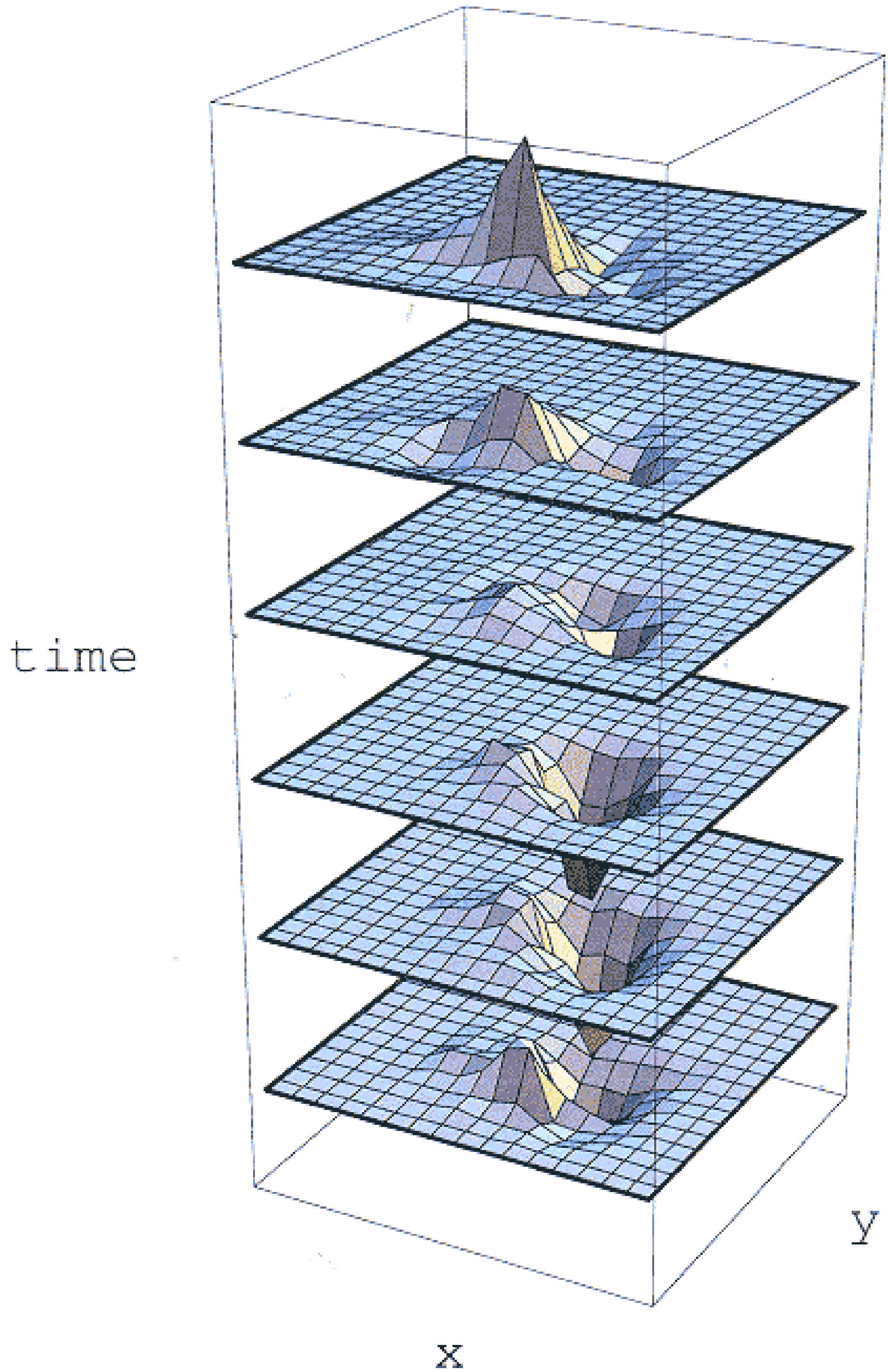
$\langle \bar{q} q \rangle \sim$ density of Dirac eigenvalues close to zero



Picture of zero-mode in classical field theory



A real QCD zero mode in a $16^3 \times 6$ lattice.


time
y
x

The collaboration

QCDSF

in Regensburg	outside
M. Göckeler *	Schierholz (DESY Zeuth)
P. Rakow *	Schiller (Leipzig)
H. Hehl	Horsley (Edinb...)
S. Schaefer	⋮
	(+ UKQCD)

chiral fermions

Ch. Gattringer *	P. Hasenfratz (Bern)
R. Hoffmann	F. Niedermayer (Bern)
W. Söldner	⋮
	Ch. Lang (Graz)

Conclusions:

- Lattice calculations provide a consistent picture of hadron structure

- The accuracy for moments of distribution functions is often not better than 30%

- Lattice calculations can be systematically improved

now

dynamical quarks

chiral quarks

chiral extrapolation
$m_q \to 0$

⋮

in reach

?
2010
?

- the instanton liquid model looks fine